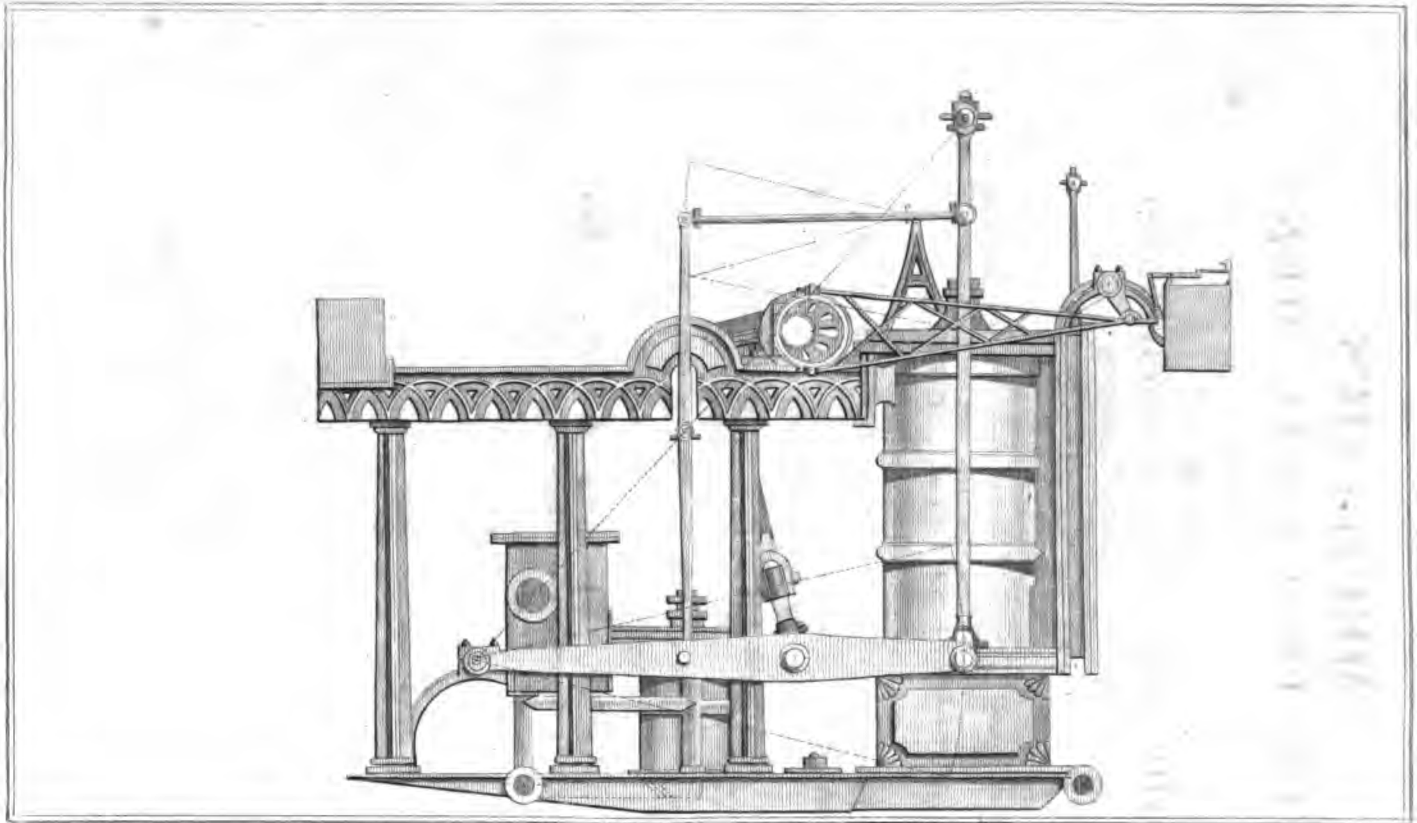

L. Dodds Del.

ELEVATION OF A STEAM BOAT ENGINE

MECHANICS
FOR PRACTICAL MEN:

CONTAINING

EXPLANATIONS OF

THE PRINCIPLES OF MECHANICS;

THE STEAM ENGINE,

WITH ITS VARIOUS PROPORTIONS;

PARALLEL MOTION,

THE PRINCIPLES OF WHICH ARE FULLY AND CLEARLY INVESTIGATED, WITH PRACTICAL RULES, ADAPTED TO THE COMMONEST CAPACITY;

TABLES OF SAFETY-VALVE LEVERS;

TABLES OF PARALLEL MOTIONS;

TABLES OF THE WEIGHT OF CAST IRON PIPES;

Tables of various Kinds, on Cast and Wrought Iron, for the Use of Founders, Smiths, &c.;

Strength and Stress of Materials;

CENTRES OF GRAVITY, GYRATION, &c.;

CENTRAL FORCES,

With their Application to the Theory of Fly Wheels, &c.;

HYDROSTATICS, AND HYDRAULICS.

WITH A SHORT DISSERTATION ON RAIL-ROADS, &c.

BY

JAMES HANN, TEACHER OF MATHEMATICS,

AND

ISAAC DODDS, CIVIL ENGINEER.

The labour spent on books or arts
Is both a waste of time and parts,
If not directed to that end
On which man's worth and weal depend.

Newcastle upon Tyne:

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1833.

TO

THE RIGHT HONOURABLE

HENRY

LORD BROUGHAM AND VAUX,

LORD HIGH CHANCELLOR

OF

Great Britain.

We beg Leave to inscribe this little Work to your Lordship, as a humble Tribute of Gratitude for your able, determined, and persevering Efforts, both to promote the Education and to ameliorate the Condition of your Fellow Men; and that your Lordship may long live to enjoy that Happiness and Peace of Mind which ever attend a Life spent in the Advancement of Science and the Improvement of our Species, is the ardent and sincere Wish of

The Authors.

Heworth Shore,
January 12, 1833.

PREFACE.

MECHANICAL SCIENCE is, beyond all doubt, one of the most useful branches of education which any age, or any country, can boast of. It is mechanical science that has been chiefly instrumental in raising England to that proud and enviable situation which she now holds in the scale of nations, and in making her the emporium of the trade of the whole world. Any attempt, therefore, to simplify or add to our stock of knowledge on this branch, however unsuccessful, may be freely pardoned.

The work which we now lay before the public consists principally of three parts. The first part is both theoretical and practical, and contains the elementary principles of Statics and Dynamics, in which the articles are all regularly numbered; for we considered this necessary in point of reference, as the propositions have a dependance upon each other. But the other parts being mostly practical, we did not consider such an arrangement necessary in them. In the parts on falling bodies, &c. we have given practical rules; and each example is done both by the formulæ and by the rules which are deduced from them. On account of the great usefulness of algebraic formulæ, we would advise every one to make himself expert in substituting numbers for the letters; and this may be very easily effected, even by those who know nothing of algebra except the signs and the notation.

The second part contains an account of the Steam-Engine, with its various proportions, in which is given a more extensive view of Parallel Motion than is to be found in almost any other work. This is a subject which has not only very much perplexed the practical mechanic, but also many whose scientific acquirements are considerably above mediocrity. The learned

Professor Millington, at page 289 of his Epitome of Natural Philosophy, has given a method for finding the length of the radius rod, which may easily mislead the practical mechanic. His method is to make the radius rod always equal to A B, (see Fig. 5 to Steam Engine) which will only hold when the point B is in the middle of A C', or, which is the same, when A B = B C', and the end of the radius rod is put on at D; for if it be put on at any other point E or *e*, either above or below the point D, it will cease to hold in this case also. But the learned professor seems not to have considered this subject with that attention which is necessary; for he says that the point D will describe a right-lined motion, which it transmits to C; but this is evidently false, for since one end of the radius rod is fixed, the other end D will evidently describe a circle. Also, we have demonstrably proved that the point F, and not the point D, describes the same kind of a line as the point C, where the piston rod is attached.

A knowledge of the proportional friction of cylinders is useful to the mechanic, though not mentioned in the body of the work. The proportion which the friction of a large cylinder bears to the friction of any number, the sum of the areas of which is equal to the area of the large cylinder, may be easily shewn as follows. Let d = the diameter of one of the small cylinders, and n = the number of them; then $d^2 n$ = area of the large one in circular inches; and $\sqrt{d^2 n} = d \sqrt{n}$ = the diameter of the large cylinder. But the friction is proportional to the circumference of the cylinder; therefore the friction of the small cylinders may be represented by $3{\cdot}1416\, d\, n$, and the friction of the large cylinder may be represented by $3{\cdot}1416\, d \sqrt{n}$. Hence the friction of the large cylinder : to the friction of all the small ones :: $3{\cdot}1416\, d \sqrt{n} : 3{\cdot}1416\, d\, n :: \sqrt{n} : n$, which, in the case of four small cylinders, becomes 1 to 2; that is, the friction of four cylinders is double the friction of one cylinder, the area of which is equal to the sum of the areas of all the four.

The third part contains the first principles of Hydrostatics and Hydraulics, the Specific Gravities of bodies, and an extensive collection of problems to elucidate these subjects; at the end of which is added a short dissertation on Rail-roads. But we would refer those who wish to have a profound knowledge on this sub-

ject to Mr. Wood's excellent practical Treatise on Rail-roads; also, much valuable information may be had by consulting Mr. Tredgold's work on the same subject. We have appended tables of various kinds, viz. a table of areas of circles, from 1 inch diameter up to 80 inches;—parallel motion tables, in calculating which we were assisted by Mr. Robert Milburn, engineer, Friars Goose, who, by an ingenious contrivance, tried a great number of them experimentally, none of which ever deviated sensibly from a vertical straight line;—also tables on cast and wrought iron, with an example to each to shew how they are calculated. We have given, lastly, a table of the diameters of piston rods for high pressure engines, pressure from 20 to 50 *lbs.* per square inch: these tables are calculated at upwards of double pressure.

The works which we have consulted on Mechanics are those of Dr. Gregory, and his edition of Hutton's Course; also the works of Whewell, Bridge, Marrat, and Emerson. The articles on Central Forces are principally taken from Gregory's Mathematics for Practical Men. Those parts which are taken from the invaluable works of Barlow and Tredgold are noticed in the body of the work. We have omitted all those parts on the forms of the teeth of wheels, considering that for want of room we would not be able to do justice to them. On this subject, see the works of Gregory, Buchanan, Camus, &c.

In conclusion, as we have occasionally pointed out the errors of other writers, we sincerely hope that our readers will deal as candidly with any which may have inadvertently crept into this work; and they may rest assured that we will cheerfully acquiesce in the discovery of our blunders, for it is not our wish to mislead the practical mechanic by that which is false, clothed in the garb of truth.

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MECHANICS

FOR PRACTICAL MEN.

MECHANICAL DEFINITIONS.

1. MECHANICS is a science which teaches the proportion of the forces, motions, velocities, and, in general, the actions of bodies upon one another.

2. Body is the mass, or quantity of matter, in any material substance; and it is always proportional to its weight or gravity, whatever its figure may be.

Body is either hard, soft, or elastic. A hard body is such that its parts do not yield to any stroke or percussion, but retain their figure unaltered. A soft body is such that its parts yield to any stroke or impression without restoring themselves again, the figure of the body remaining altered. And an elastic body is such that the parts will yield to any stroke, but which presently restore themselves again, and the body regains the same figure as before the stroke.

There are no bodies that are perfectly hard, soft, or elastic; but all partaking these properties, more or less, in some intermediate degree.

3. Bodies are either solid or fluid. A solid body is such that its parts are not easily moved among one another, and which retains any figure given to it. But a fluid body is such that the parts yield to the slightest impression, being easily moved among one another; and its surface, when left to itself, is always observed to settle in a smooth plane at the top.

4. Density of a body is the proportion of the quantity of matter contained in it, to the quantity of matter contained in another body of the same size or magnitude. Thus, the density is said to be double or triple, when the quantity of matter contained in the same space is double or triple.

5. Force is a power exerted on a body to move it. If it act but for a moment, it is called the force of percussion or impulse. If it act constantly, it is called an accelerative force. If constantly and equally, it is called an uniform accelerative force.

6. Velocity is an affection of motion, by which a body passes over a certain space in a certain time. Thus, if a body in motion pass uniformly over 10 feet in 2 seconds of time, it is said to move with the velocity of 5 feet per second; and so on.

7. Motion is a continual and successive change of place. If a body moves through equal spaces in equal times, it is called equable motion. If its velocity continually increases, it is called accelerated motion. If it decreases, it is retarded motion. If it increases or decreases uniformly, it is equably accelerated or retarded. Likewise if its motion be considered in regard to some other body at rest, it is called absolute motion; but if its motion be considered with respect to other bodies also in motion, then it is relative motion.

8. Direction of motion is the way the body tends, or the right line it moves in.

9. Momentum, or Quantity of Motion, is the power or force in moving bodies by which they continually tend

from their present places, or with which they strike any obstacle that opposes their motion.

10. Forces are distinguished into Motive, or Accelerative or Retardive. A Motive, or moving force, is the power of an agent to produce motion; and it is equal or proportional to the momentum it will generate in a body, when acting either by percussion, or for a certain time as a permanent force.

11. Accelerative or Retardive Force is commonly understood to be that which affects the velocity only, or it is that by which the velocity is accelerated or retarded; and it is equal or proportional to the motive force directly, and to the mass or body moved inversely. So that if a body of 3 pounds weight be acted on by a motive force of 6, then the accelerative force is $\frac{6}{3} = 2$; but if the same force, 6, act on another body of 6 pounds weight, then the accelerating force in this case is $\frac{6}{6} = 1$, and so is but half the former, and will produce only half the velocity.

12. Gravity is that force wherewith a body endeavours to descend towards the centre of the earth. This is called Absolute Gravity when the body tends downwards in free space; and Relative Gravity is the force it endeavours to descend with in a fluid.

13. Specific Gravity is the relation of the weights of different bodies of equal magnitude, and so is proportional to the density of the body.

14. Centre of Gravity of a body is a certain point in it, upon which, the body being freely suspended, it would rest in any position.

15. Centre of Motion of a body is a fixed point about which the body is moved; and the Axis of Motion is the fixed axis it moves about.

16. Weight and Power, when opposed to one another, signify the body to be removed, and the body that moves it. That body which communicates the motion is called the power; and that which receives it, the weight.

17. Statics has for its object the equilibrium of forces applied to solid bodies.

18. By Dynamics* we investigate the circumstances of the motion of solid bodies.

19. Hydrostatics is the science in which the equilibrium of fluids is considered.

20. Hydraulics is the art of raising or conveying water by the help of engines.

21. Pneumatics relates to the properties of elastic fluids, and, according to Def. 19, is a branch of Hydrostatics.

LAWS OF MOTION.

22. First, A body must continue for ever in a state of rest, or in a state of uniform and rectilineal motion, if it be not disturbed by the action of some external cause.

23. Second, The alteration of motion produced in a body by the action of any external force, is always proportional to that force, and in the direction of the right line in which it acts.

24. Third, The action and re-action of bodies on one another are equal, and are exerted in opposite directions.

The third law of motion involves two distinct propositions:—

1st, When motion is communicated by impulse, the quantity of motion gained by the one body in any direc-

* The term Dynamics signifies literally the doctrine of power; power or force being known to us only as the cause of motion, and measured by the motion it produces.

tion, is just equal to that which is lost by the other in the same direction.

2d, When motion is communicated without apparent contact, as in the case of gravitation, and of the phenomena ascribed to attraction or repulsion, the quantity of motion gained by the one body is just equal to that which is gained by the other, but in an opposite direction. Thus, if any two bodies or masses, whatever may be their natures, be placed at rest in free space, at any proposed distance from each other, and beyond the influence of every other body, they will immediately begin to approach each other, but not with a uniform velocity, but with a velocity which is continually accelerated. If the bodies are equal, they will approach each other with equal velocities; but if they are unequal, the velocity of the lesser will be greater than that of the greater in the same proportion as its mass is less; therefore the quantity of motion gained by the one body is just equal to the quantity of motion gained by the other, but in an opposite direction.

Magnetic Attraction is of a similar nature: but while it acts only between some particular bodies, Gravitation acts upon all bodies, of all species or kinds, and under all circumstances; it being a force the intensity of which is totally independent of the nature of the bodies, and only depends on their masses and mutual distances; and, from its universality, it has sometimes been called the Law of Nature, or Universal Gravitation.

STATICS.

COMPOSITION AND RESOLUTION OF FORCES.

25. The Composition of Forces is the operation by which the resultant of any number of forces, applied to the same point or body, is determined.

26. The Resolution of Forces is just the inverse of the composition; and it is sometimes called the Decomposition of Forces.

27. If two forces act upon a body in the same direction, the combined effect is equivalent to the sum of the forces. Thus, if a force of 4 pounds, and another of 5 pounds, act upon the same body in the same direction, the body will be acted on, or drawn, by a force of 9 pounds. But if two forces act in opposite directions, the resultant will be the difference of the two, and in the direction of the greater. Thus, if a body be acted on by a force of 8 pounds in one direction, and 6 pounds in the opposite, it will be drawn by a force of 2 pounds in the direction of the greatest force.

28. If a body at A (Plate I. Fig. 1) be acted upon separately by two forces in the directions A B and A C, which would cause the body to be carried through the spaces A B and A C in the same time, then both forces acting together will cause the body to describe the diagonal A D of the parallelogram A B C D, in the same time in which it would describe either of the sides by either of the forces acting separately.

If the forces at A cause the body to move uniformly along the lines A B, A C, then, since the force acting in direction A C parallel to B D, by the second law of motion, will not alter the velocity of the body towards the line B D, the body will therefore arrive at B D in the

same time, whether the force in direction A C be impressed or not; therefore, at the end of the time, it will be found somewhere in the line B D. By the same argument, it will be found somewhere in the line C D: therefore it will be found in D, their point of intersection; and, by the first law of motion, it will move in a right line from A to D.

Otherwise,

Suppose the line A C to move parallel to itself into the place B D, whilst A moves from A to C, then, since this line and the body are both equally moved towards B D, it is evident that the body will always be in the moveable line A C: therefore, since the motions are uniform, and the lines A B, A C, are described in the same time, the body will describe the straight line A D, the diagonal of the parallelogram.

But if the body A (Fig. 2) be carried through A B by a uniform force in the same time that it would be carried through A C by an accelerative force, then, by both forces acting together, it will, at the end of that time, be found in the point D, as before; but, in this case, it will describe a curve line A F D.

Cor. 1. The forces in the directions A B, A C, A D, are respectively proportional to the lines A B, A C, A D. (Fig. 1.)

Cor. 2. The two oblique forces A B and A C, are equivalent to the single and direct force A D, which is compounded of these two, by drawing the diagonal of the parallelogram. (Fig. 3.)

Cor. 3. If two forces, as A B and A C, act in directions A B and A C respectively, then, because the diagonals of a parallelogram bisect each other, the forces represented by A B and A C are equivalent to twice the force represented by A E.

Cor. 4. A given force may be resolved into two others, one of which is given and has a given direction.

29. If a body be kept in equilibrio by the joint action of three forces in the same plane, these forces will be respectively proportional to the three sides, A B, B C, A C, of a triangle, which are drawn parallel to the directions of the forces D A, E A, A C. (Fig. 4.)

Let A C represent the force C, and produce D A, E A, and complete the parallelogram A B C F. Now, by the last Prop. the force A C is equivalent to the two forces A B, A F; put, therefore, the forces A B, A F, instead of A C, and all the forces will still be in equilibrio: therefore, since A C represents the force C, then A B will represent its opposite force D, and B C or A F its opposite force E. Consequently, the three forces D, E, C, are proportional to A B, B C, A C, the three sides of the triangle A B C, formed by drawing lines parallel to the directions of the three forces.

Cor. 1. The three forces D, E, C, will be respectively as the sines of the angles A C B, C A B, A B C; for these forces are as A B, B C, A C, and these sides are as the sines of their opposite angles C, A, B.

Cor. 2. Three forces acting upon a body, and keeping it in equilibrium, are proportional to the sides of a triangle formed by drawing lines either perpendicular to the directions in which the forces act, or making any given angles with those directions. For such a triangle is always similar to that which is made by drawing lines parallel to the directions.

30. If three forces, the directions of which concur in one point, are represented by the three contiguous edges of a parallelopiped, their resultant will be represented, both in magnitude and direction, by the diagonal drawn from the point of concourse to the opposite angle of the parallelopiped.

Let the directions in which the forces act be A B, A C, A D, and A G, (Fig. 5) and complete the parallelopiped B F. Then, since A B H C is a parallelogram, the force

$P \times AC + Q \times BC + R \times DC = S \times EC + T \times FC$.

Cor. 9. Since $P : W :: CW : AC$, we have, by composition, (Fig. 16)

$P + W : P :: AW : CW$.

and $P + W : W :: AW : AC$.

$$\therefore CW = \frac{P \times AW}{P + W} \text{ and } AC = \frac{W \times AW}{P + W}$$

This corollary is useful in finding the fulcrum, when the power and weight are both given, together with the whole length of the lever.

Or it may be done very simply by Algebra. Thus, let AW the whole length of the lever $= a$, and $CW = x$; then $AC = a - x$, and by Cor. 5.

$W x = P(a - x) = P a - P x$,

$P x + W x = P a$

$$\therefore x = \frac{P a}{P + W} = \frac{P \times AW}{P + W}, \text{ as before.}$$

41. In the compound lever, or where several levers act perpendicularly upon one another, as AB, BC, CD (Fig. 17), the fulcrums of which are F, G, I; then $P : W :: BF \times CG \times DI : AF \times BG \times IC$.

For the power P acting at A : weight at $B :: BF : AF$, and the force or power at B : the weight at $C :: CG : BG$.

Also the power at C : the weight $W :: DI : IC$.

Therefore, ex equo, $P : W :: BF \times CG \times DI : AF \times BG \times IC$.

Cor. The pressure on the fulcrum $F = P + Q = \frac{P \times AB}{BF}$ by Cor. 9 to the last Art.

The pressure on the fulcrum $G = Q + R = \frac{P \times AF}{BF} + \frac{W \times ID}{CI}$

Upon $I = R + W = W + \frac{W \times ID}{CI} = \frac{W \times CD}{CI}$

42. The following rule holds, whether the lever be of the first, second, or third order.

Multiply the power by its distance from the fulcrum, and this product will be equal to the weight multiplied by its distance from the fulcrum, the weight of the lever not being considered.

43. If the weight upon each end of the lever be given, to find the fulcrum:

Then the sum of the weights, is to either of the weights, as the sum of the distances, or whole length of the lever, is to the distance of the other.

The Balance.

44. The Balance is a lever with equal arms, and, when well constructed, must have the following properties:

1. The points of suspension of the scales, and the centre of motion of the beam, A, C, B, (Fig. 18) should be in a straight line. 2. The arms, A C, B C, must be of an equal length. 3. That the centre of gravity be in the centre of motion C, or a little below it; for when these centres coincide, and the weights are equal, the addition of the smallest weight would overset the balance, and place it in a vertical situation, from which it would have no tendency to return. The sensibility, in this case, would be the greatest possible; but the requisites of level and stability would be entirely lost.* 4. That they be in

* If a be the length of the arm of the balance, and b the distance between the centre of suspension and centre of gravity, P the load in either scale, and W the weight of the beam, the sensibility of the balance is as $\frac{a}{b\,(2\,P + W)}$. It is therefore greater, the greater the length of the arm, the less the distance between the two centres, and the less the weight with which the balance is loaded.

The stability, or the force with which the state of equilibrium is recovered, is proportional to $(2\,P + W)\,b$, the denominator of the preceding fraction. The diminution of b, therefore,

equilibrio when empty. 5. That there be as little friction as possible at the centre C.

The False Balance.

45. The false balance has its arms of an unequal length, and is in equilibrium when loaded with unequal weights. To find the true weight of any body by a false balance, First, weigh the body in one scale, and afterwards weigh it in the other; then the mean proportional between these two weights will be the true weight required. For let x = the true weight of the body, and W the number of pounds or ounces it weighs in the scale D, and only w the pounds or ounces in the scale E; then we have these two equations, $AC \cdot x = BC \cdot W$,

and $BC \cdot x = AC \cdot w$.

Multiply these equations together, and $AC \cdot BC \cdot x^2 = AC \cdot BC \cdot W w$

$\therefore x^2 = W w$ or $x = \pm \sqrt{W w}$.

That is, the true weight is a mean proportional between the weights W and w.

Construction of the Steelyard.

46. The steelyard A B (Fig. 19) is a lever of the first order, the fulcrum or centre of motion of which is C. If the weight P, placed at D, just balances the weight of the long arm A C, or reduces the beam A B to an equilibrium, and there be taken the equal divisions D 1, 1 2, 2 3, 3 4,

while it increases the sensibility, diminishes or lessens the stability of the balance. The lengthening of a will, however, increase the former of these quantities, without diminishing the latter.

The above formulas are of great practical utility; because, by means of them, one balance may be made, having exactly the same sensibility and stability with another. It is only required that the ratio of the length of the arms should be the same with that which is compounded of the ratios of the distances of the centres of gravity and suspension, and of the weights of the beams.—*Playfair's Outlines of Natural Philosophy.*

&c.; then the weight P, placed successively at 1, 2, 3, 4, &c. will balance weights as W, suspended at B, which are also as the numbers 1, 2, 3, 4, &c. respectively. Also if the divisions D 1, 1 2, 2 3, &c. be each equal C B, then if P be successively placed at 1, 2, 3, &c. the weight W to balance it will be respectively equal to P, 2 P, 3 P, &c. that is to 1, 2, 3 pounds, &c. if P is one pound. For by the property of the lever, $C P \times P + C D \times P = B C \times W$; that is, $P D \times P = B C \times W$. And $B C : P D :: P : W$. Therefore if D P or $D 1 = B C$, then $W = P$. If D P or $D 2 = 2 B C$, then $W = 2 P$, &c. But if B C be greater than D 1, 1 2, &c. then will the constant weight P be greater than W, 2 W, &c.

47. If we take the weight of the lever into our investigation, we must consider its whole weight to act at its centre of gravity; and if the lever be in the form of a cylinder, prism, or an uniform bar of any kind, its centre of gravity will be in its middle point.

We will show afterwards how to take the weight of the lever into consideration, when the centre of gravity is not in the middle point of the lever.

In a lever of the first order, we may consider the weight of each arm of the lever as a new power acting at its centre of gravity, therefore, (see Fig. 12)

$P \times A C$ + weight of $A C \times \frac{1}{2} A C = W \times C W$ + weight of $C W \times \frac{1}{2} C W$.

In a lever of the second kind, we have,*

$P \times A C = W \times C W$ + weight of $A C \times \frac{1}{2} A C$. (Fig. 13.)

* When a beam carrying a weight is supported in a horizontal position by two props or posts, the weights which the props sustain are inversely proportional to their distances from the centre of gravity of the weight.

Let p = pressure at A, and P = pressure at C, (see Fig. 24)

$$\text{then } p : W :: C W : A C$$
$$W : P :: A C : A W$$
$$\therefore \text{ex æquo } p : P :: C W : A W$$

In a lever of the third order (Fig. 14),

$P \times A\,C = W \times C\,W +$ weight of $C\,W \times \frac{1}{2}\,C\,W$.

Example 1.

Let the part A C = 5 feet, its weight $2\frac{1}{2}$ *lbs.* the part C W = 3 feet, and its weight $1\frac{1}{2}$ *lb.*; and if 100 *lbs.* is suspended at W, what weight must be put on at A, that the lever may be in equilibrio? (See Fig. 12.)

The lever being uniform, the centre of gravity of A C is $2\frac{1}{2}$ feet from the fulcrum; in like manner, the centre of gravity of C W is $1\frac{1}{2}$ feet from the same point; or, instead of considering the whole weight of each part as acting at the centre of gravity, we may consider half the weight acting at the end, and the results will be exactly the same. For the whole weight of the part A C, acting with the leverage of $\frac{1}{2}$ A C, is exactly the same as half that weight acting with the whole leverage A C. The same reasoning manifestly holds for the part C W.

Therefore 5 feet $\times 1\frac{1}{4}$ *lb.* $= 6{\cdot}25$ *lbs.* from the action of the part A C of the lever;

and 3 feet $\times {\cdot}75$ *lb.* $= 2{\cdot}25$ *lbs.* from the action of the part C W;

and C W $\times 100 = 3 \times 100 = 300$ *lbs.* and $300 + 2{\cdot}25 = 302{\cdot}25$, the effect produced by the action of both the weight and the weight of the part C W of the lever; and this must be equal to the weight put on at A, multiplied by its distance from the fulcrum, together with the weight produced from the action of the part A C of the lever, so as to obtain an equilibrium.

Hence, the weight required must be equal to $\frac{302{\cdot}25 - 6.25}{5} = 59\frac{1}{5}$ *lbs.*

Or thus, by Art. 47, $P \times A\,C +$ weight $A\,C \times \frac{1}{2}\,A\,C = W \times C\,W +$ weight $C\,W \times \frac{1}{2}\,C\,W$; that is, $P \times 5 + 2\frac{1}{2}$ *lbs.* $\times 2\frac{1}{2}$ feet $= 100 \times 3 + 1\frac{1}{2}$ *lbs.* $1\frac{1}{2}$ feet; or, $P = \frac{300 + 2\frac{1}{4} - 6\frac{1}{4}}{5} = 59\frac{1}{5}$ as above.

Example 2.

Given the whole weight of the lever 42 *lbs.* the length of the part A C = 8 feet, and the length of the part C W = 6 feet; a weight of 3 *cwt.* is suspended at 2 feet from C towards W; it is required to determine what weight must be put on at 5 feet from C towards A, to produce an equilibrium?

$\frac{42}{14} = 3$ *lbs.* the weight of one foot of the lever.

Therefore $8 \times 3 = 24$ *lbs.* the weight of the part A C.

$6 \times 3 = 18$ *lbs.* the weight of the part C W.

The centre of gravity of C W is 3 feet from F, and the centre of gravity of A C is 4 feet from C; consequently, 18 *lbs.* $\times\ 3 = 54$ *lbs.* the effect produced from the action of the part C W of the lever; and $24 \times 4 = 96$ *lbs.* the effect produced by the action of the part A C.

And 3 *cwt.* = 336 *lbs.* and $336 \times 2 = 672$ *lbs.*; therefore $672 + 54 = 726$ *lbs.* the effect produced from the action of both weight and lever; hence $\frac{726 - 96}{5} =$ 126 *lbs.* the weight required.

48. From what has been already premised, we may deduce formulæ for the lever of the first order, when the power acts at one end of it, and the weight at the other.

Put A C = a, and C W = b, and let the weight of one inch in length of the lever = c, P the power, and W the weight.

Then $Pa + \frac{1}{2}a^2c = Wb + \frac{1}{2}b^2c$, and this equation solved for P gives $P = \frac{Wb + \frac{1}{2}b^2c - \frac{1}{2}a^2c}{a}$. (1)

And if the same equation be solved for W, we have,

$$W = \frac{Pa + \frac{1}{2}a^2c - \frac{1}{2}b^2c}{b}. \quad (2)$$

In the same manner, we have,

$$a = \frac{1}{c}\sqrt{(P^2 + 2\,W\,b\,c + b^2\,c^2)} - \frac{P}{c}. \quad (3)$$

$$b = \frac{1}{c}\sqrt{(W^2 + 2\,P\,a\,c + a^2\,c^2)} - \frac{W}{c} \quad (4)$$

$$c = \frac{2\,P\,a \sim 2\,W\,b}{b^2 \sim a^2}. \quad (5)$$

When $W = O$, we have,

$$a = \sqrt{\left(b^2 + \frac{P^2}{c^2}\right)} - \frac{P}{c}. \quad (6)$$ From equation 3.

And when $P = O$,

$$a = \sqrt{\left(b^2 + \frac{2\,W\,b}{c}\right)}. \quad (7)$$ By making $P = O$ in equation (3).

49. We will now proceed to deduce formulæ for the case considered in Example 2d, where the power acts at some intermediate point between A and C, and the weight acts also at some intermediate point between W and C.

Let, as before, $P =$ the power and W the weight (Fig. 20); also, $A\,C = a$, $C\,W = b$, and the weight of one inch in length $= c$; and put $C\,D = d$, the distance from the fulcrum that the power acts, and $C\,E = r$, the distance from the fulcrum that the weight acts.

Then, $P\,d + \frac{1}{2}\,a^2c = W\,r + \frac{1}{2}\,b^2c$, and this, solved for P, gives,

$$P = \frac{W\,r + \frac{1}{2}\,b^2c - \frac{1}{2}\,a^2c}{d}. \quad (8)$$

$$W = \frac{P\,d + \frac{1}{2}\,a^2c - \frac{1}{2}\,b^2c}{r}. \quad (9)$$

$$d = \frac{2\,W\,r + b^2c - a^2c}{2\,P}. \quad (10)$$

$$r = \frac{2\,P\,d + a^2c - b^2c}{2\,W}. \quad (11)$$

And c is the same as in formula 5.

But it is more simple to divide the whole weight of the lever by the whole length, and the quotient will give the

weight of one inch or one foot in length, according as you take the length in inches or in feet.

To apply the formulæ we have deduced, we may take Example 1.

Now, in this case, the power acts at one end of the lever, and the weight at the other, and the power is required; therefore formula (1) is exactly adapted to this case.

$$\text{That is, } P = \frac{W\,b + \frac{1}{2}\,b^2c - \frac{1}{2}\,a^2c}{a}$$

Here W = 100 *lbs.* $a = 60$ inches, $b = 36$, and $c = \frac{4}{96} = \frac{1}{24}$ *lb.*; therefore,

$$P = \frac{100 \times 36 + \frac{1}{2} \times 36^2 \times \frac{1}{24} - \frac{1}{2} \times 60^2 \times \frac{1}{24}}{60} =$$

$\frac{3600 + 27 - 75}{60} = 59{\cdot}2$ *lbs.* the same as before in Ex. 1.

But if we take c for the weight of one foot in length, it becomes $P = \frac{100 \times 3 + \frac{1}{2} \times 3^2 \times \frac{1}{2} - \frac{1}{2} \times 5^2 \times \frac{1}{2}}{5} =$

$\frac{300 + 2{\cdot}25 - 6.25}{5} = 59{\cdot}2$, as above.

We will now take Example 2d (Fig. 20), where the power is applied at a given point D between A and C, and the weight at a given point E between W and C; and since the power P is required, we must take formula (8).

$$P = \frac{W\,r + \frac{1}{2}\,b^2c - \frac{1}{2}\,a^2c}{d}$$

Here W = 3 *cwt.* = 336 *lbs.* $a = 8$ feet, $b = 6$ feet, $c = \frac{42}{14} = 3$ *lbs.* weight of one foot of the lever, $d = 5$ feet, and $r = 2$ feet; substitute these values in the above, and P = $\frac{336 \times 2 + \frac{1}{2} \times 6^2 \times 3 - \frac{1}{2}\,8^2 \times 3}{5} = \frac{672 + 54 - 96}{5} =$

126 *lbs.* which is exactly the same as was found in Example 2d.

50. Let A W C be a lever of the second order, and C its fulcrum; the power multiplied by its distance from the fulcrum, is equal to the weight multiplied by its distance

from the fulcrum, together with the whole weight of the lever multiplied by half its length; the lever being considered uniform throughout its length.

Example 1.

Given the whole weight of the lever 9 *lbs.* its length A C = 6 feet, a weight of 100 *lbs.* is put on at $1\frac{1}{2}$ feet from the fulcrum; it is required to determine the power acting at A which will keep the lever in equilibrio. (See Fig. 13.)

Now 100 *lbs.* $\times$ $1\frac{1}{2}$ feet = 150 = the weight multiplied by its distance.

$9 \times 3 = 27$ = the weight of the lever multiplied by half its length.

Hence $\frac{150 + 27}{6} = 29\frac{1}{2}$ *lbs.* is the weight or power acting at A which will keep the whole in equilibrio.

Or thus, by note to Art. 47:

$6 : 100 :: 4\frac{1}{2} : 75$, the weight upon the fulcrum from the action of the weight.

$6 : 100 :: 1\frac{1}{2} : 25$, the power at A which will just support the weight.

And the lever being uniform, its whole weight must be considered as acting at the middle of A F; therefore the fulcrum will bear one half of its weight, and the power must support the other.

Consequently $75 + 4\frac{1}{2} = 79\frac{1}{2}$ *lbs.* weight upon the fulcrum.

And $25 + 4\frac{1}{2} = 29\frac{1}{2}$ *lbs.* the power necessary to keep the whole in equilibrio, which is exactly the same as before.

Example 2.

A beam, the weight of which is 12 *lbs.* and its length 18 feet, is supported at both ends; a weight of 36 *lbs.* is suspended at 3 feet from one end, and a weight of 24 *lbs.* at 8 feet from the other end; required the pressure on each prop or support. (Fig. 21.)

For the sake of simplicity, suppose the 36 *lbs.* weight to be suspended at 3 feet from the end C, and the 24 *lbs.* at 8 feet from the end A.

Then, 18 : 36 :: 15 : 30 *lbs.* the pressure on the support C by the action of the 36 *lbs.* weight.

18 : 24 :: 8 : 10⅔ *lbs.* the pressure on the support C from the action of the 24 *lbs.* weight.

30 + 10⅔ = 40⅔ *lbs.* pressure on the support C from both weights.

And 18 : 36 :: 3 : 6 *lbs.* pressure on the support A from the action of the 36 *lbs.* weight.

18 : 24 :: 10 : 13⅓ *lbs.* pressure on the support A from the 24 *lbs.* weight.

6 + 13⅓ = 19⅓ *lbs.* the whole pressure on the support A from both weights.

Now half the weight of the lever added to each of the above sums will give the whole pressure on each support.

Thus, 40⅔ + 6 = 46⅔ *lbs.* whole pressure on the support C.

And 19⅓ + 6 = 25⅓ *lbs.* the whole pressure on the support A.

Example 3.

Given the whole length of the lever A C = 10 feet (Fig. 22), its weight 15 *lbs.* a weight of 50 *lbs.* is suspended at 2 feet from the fulcrum or end C; what power, acting at 3 feet from the other end A, will keep the whole in equilibrio?

Now 10 — 3 = 7 feet, the distance of the power from the fulcrum C; therefore 7 : 50 :: 2 : 14$\frac{2}{7}$ *lbs.* the power necessary to balance the weight alone; and since the centre of gravity of the lever is 2 feet from the power, and 5 feet from the fulcrum, we have

7 : 15 :: 5 : 10$\frac{5}{7}$ *lbs.* the power necessary to sustain the lever.

And 14$\frac{2}{7}$ + 10$\frac{5}{7}$ = 25 *lbs.* the power required to sustain both weight and lever.

In the same manner we may find the weight sustained by the fulcrum.

Thus, $7 : 50 :: 5 : 35\frac{5}{7}$ *lbs.* from the action of the weight.

And $7 : 15 :: 2 : 4\frac{2}{7}$ *lbs.* from the action of the lever,

Therefore $35\frac{5}{7} + 4\frac{2}{7} = 40$ *lbs.* the whole pressure on the fulcrum or end C.

51. We may now deduce formulæ for the lever of the second order, retaining the same notation as in Art. 48.

We have $P a = W b + \frac{1}{2} a^2 c$, from which we have

$$P = \frac{W b}{a} + \frac{a c}{2}. \quad (1)$$

$$W = \frac{P a - \frac{1}{2} a^2 c}{b}. \quad (2)$$

$$a = \frac{1}{c}\left(P \pm \sqrt{(P^2 - 2 W b c)}\right) \quad (3)$$

$$b = \frac{P a - \frac{1}{2} a^2 c}{W}. \quad (4)$$

$$c = \frac{2 P a - 2 W b}{a^2}. \quad (5)$$

When $W = O$, or the power just sustains the lever, we have $P = \frac{1}{2} a c$, and $a = \frac{2 P}{c}$. (6)

If $P = O$, the expression for a is impossible; therefore the equation for P admits of a minimum; and by Dr. Gregory's Mechanics, Art. on the Lever, when P is a minimum, $a = \sqrt{\frac{2 W b}{c}}$

Take Example 1; and in that case $W = 100$, $a = 6$, $b = 1\frac{1}{2}$, and $c = 1\frac{1}{2}$; consequently we have, by formula (1)

$$P = \frac{100 \times 1\frac{1}{2}}{6} + \frac{6 \times 1\frac{1}{2}}{2} = \frac{150}{6} + \frac{9}{2} = 29\tfrac{1}{2} \textit{ lbs.}$$

We will here give an example when a beam is supported by two posts or props, neither of which are at the ends of the beam.

Example 4.

A beam, the weight of which is 84 *lbs.* and its length A B = 20 feet, is supported by two posts at C and D; the distance of the end A from C = $1\frac{1}{2}$ feet, and the distance of the end B from D = $2\frac{1}{2}$ feet; a weight of 156 *lbs.* is suspended at E, 2 feet from C; required the pressure on each post.

Now, $1\frac{1}{2} + 2\frac{1}{2} = 4 = A\,C + D\,B$, and $20 - 4 = 16 = C\,D$, the distance between the props or posts.

And 16 : 156 :: 14 : $136\frac{1}{2}$ *lbs.* pressure on the prop C from the action of the weight.

16 : 156 :: 2 : $19\frac{1}{2}$ *lbs.* pressure on the prop D from the action of the weight.

And $\frac{1}{2}$ A B = 10, and $10 - 1\frac{1}{2} = 8\frac{1}{2}$ feet, the distance of the centre of gravity from the prop C; also $10 - 2\frac{1}{2} = 7\frac{1}{2}$ feet, the distance of the centre of gravity of the beam from the prop D.

16 : 84 :: $8\frac{1}{2}$: $44\frac{5}{8}$, pressure on the prop D from the action of the lever.

16 : 84 :: $7\frac{1}{2}$: $39\frac{3}{8}$, pressure on the prop C from the action of the lever.

$136\frac{1}{2} + 39\frac{3}{8} = 175\frac{7}{8}$ *lbs.* whole pressure on the prop C.

$19\frac{1}{2} + 44\frac{5}{8} = 64\frac{1}{8}$ *lbs.* whole pressure on the prop D.

52. These examples will sufficiently explain all the different cases of levers of the second order. We will now proceed to give formulæ for the lever of the third order.

Formulæ.

The notation remaining the same, we have $P\,a = W\,b + \frac{1}{2}\,b^2c$.

Hence $P = \dfrac{W\,b + \frac{1}{2}\,b^2c}{a}$. (1)

$W = \dfrac{P\,a}{b} - \dfrac{b\,c}{2}$. (2)

$$c = \frac{2\,P\,a - 2\,W\,b}{b^2}. \quad (3)$$

$$b = \frac{1}{c}\sqrt{(W^2 + 2\,P\,a\,c)} - \frac{W}{c} \quad (4)$$

$$a = \frac{W\,b + \frac{1}{2}\,b^2 c}{P}. \quad (5)$$

When $W = O$, or the power just sustains the lever, we have $\frac{P\,a}{b} = \frac{b\,c}{2} = O$, hence $a = \frac{b^2 c}{2P}$. (6)

$$b = \sqrt{\frac{2\,P\,a}{c}}. \quad (7)$$

Formulæ for the lever of the third order, when the weight does not act at the end:

Let $r =$ the distance of the weight from the fulcrum, the rest remaining the same as in the notation of the above formulæ.

$$\text{Then } P\,a = W\,r + \tfrac{1}{2}\,b^2 c.$$

$$P = \frac{W\,r + \frac{1}{2}\,b^2 c}{a}. \quad (8)$$

$$W = \frac{P\,a - \frac{1}{2}\,b^2 c}{r}. \quad (9)$$

$$r = \frac{P\,a - \frac{1}{2}\,b^2 c}{W}. \quad (10)$$

Examples for Practice.

Example 1.

Given the whole length A W of a lever of the first order = 14 feet, a weight of 20 stones acts at W, which is $3\frac{1}{2}$ feet from the fulcrum; what power must be applied at A to produce an equilibrium?

$10\frac{1}{2} : 3\frac{1}{2} :: 20 : 6{\cdot}6$ stones.

Example 2.

In the false balance, if a body weigh 4 *lb.* in one scale, and 16 *lbs.* in the other, required the true weight of the body.

By Art. 45, $\sqrt{4 \times 16} = \sqrt{64} = 8$ *lbs.* the true weight.

Example 3.

In the bended lever, if the power P = 100 *lbs.* W = 200 *lbs.* and C A = 10 feet, C W = 5 feet, and the angle C A D = 30°; required the angle C W G, that there may be an equilibrium. (Fig. 11.)

By Art. 40, and Cor 4, we have P × C A × sine C A D = W × C W × sine C W G.

$$\therefore \text{sine C W G} = \frac{\text{P} \times \text{C A} \times \text{sine C A D}}{\text{W} \times \text{C W}}$$

Here P = 100, W = 200, C A = 10, C W = 5, and sine 30 degrees by the table of natural sines is ·5.

$$\text{Hence, sine C W G} = \frac{100 \times 10 \times \cdot 5}{200 \times 5} = \frac{500}{1000} = \cdot 5.$$

Therefore the angle C W G is also 30°.

Example 4.

Given the length of the lever for a safety-valve* 24 inches (Fig. 23), the distance between the fulcrum and the centre of the valve = A C = 3 inches, weight of the lever 4 *lbs.* and the diameter of the valve 3 inches; what weight must be put on at B, the end of the lever, to give 40 *lbs.* per square inch upon the valve?

Here $3^2 \times \cdot 7854 = 7 \cdot 0686$, area, or number of square inches in the valve; but if we take 7 square inches, it will be near enough in practice.

Then 7 × 40 = 280 *lbs.* the weight upon the whole valve; and the weight of one inch in length is $\frac{4}{24} = \frac{1}{6}$ *lbs.*

By Formula 2, in the lever of the third order,

$$W = \frac{P\,a}{b} - \frac{b\,c}{2}$$

Here P = 280 *lbs.* $a = 3$, $b = 24$, and $c = \frac{1}{6}$ *lb.*

* For the full investigation of the safety-valve lever, see the section on the Steam Engine.

$$\therefore W = \frac{280 \times 3}{24} - \frac{24 \times \frac{1}{8}}{2} = 35 - 2 = 33 \text{ lbs.}$$

That is, 33 *lbs.* put on at the end of the lever, will give 280 *lbs.* upon the whole valve, which is 40 *lbs.* per square inch.

We must now mark the lever in the points where there will be 10, 20, and 30 *lbs.* per square inch respectively.

To do this, we have the weight 33 *lbs.* the distance A C = 3 inches, and when there are 10 *lbs.* per square inch upon the valve, then $7 \times 10 = 70$ *lbs.* upon the whole valve, = P, to find what distance from the fulcrum the weight must be placed to give the above-mentioned pressure.

Here, in this case, Formula (10) is applicable.

$$r = \frac{P\,a - \frac{1}{2}\,b^2 c}{W} = \frac{70 \times 3 - \frac{1}{2} \times 24^2 \times \frac{1}{6}}{33} = \frac{210 - 48}{33}.$$

= 4·9 inches.

That is, if A D be taken = 4·9 inches, the weight of 33 *lbs.* placed at D will give 70 *lbs.* upon the whole valve, or 10 *lbs.* per square inch.

In like manner, we must find how far we must move the weight along the lever from D towards B to give 20 *lbs.* per square inch.

Here $7 \times 20 = 140$ *lbs.* upon the whole valve; therefore if we substitute 140 *lbs.* for P in the above formula, we have

$$r = \frac{140 \times 3 - \frac{1}{2} \times 24^2 \times \frac{1}{6}}{33} = 11{\cdot}27 \text{ inches} = \text{A E}.$$

Also 30 *lbs.* per square inch, multiplied by 7, gives 210 *lbs.* the weight upon the whole valve; and this put for P, we have

$$r = \frac{210 \times 3 - \frac{1}{2} \times 24^2 \times \frac{1}{6}}{33} = 17{\cdot}63 \text{ inches} = \text{A F}.$$

Therefore if you want 10, 20, 30, or 40 *lbs.* per square inch upon the valve, the weight of 33 *lbs.* must be at the distance of 4·9, 11·27, 17·63, 24 inches from the fulcrum A respectively.

The Wheel and Axle.

53. The wheel and axle consists of a wheel having a cylindric axis passing through its centre. (See Fig. 25.) The power is applied to the circumference of the wheel, and the weight to the circumference of the axle.

In the wheel and axle, an equilibrium takes place when the power multiplied by the radius of the wheel, is equal to the weight multiplied by the radius of the axle; or P : W :: C A : C B.

For the wheel and axle being nothing else but a lever so contrived as to have a continued motion about its fulcrum C, the arms of which may be represented by A C and B C, therefore, by the property of the lever, P : W :: C A : C B.

54. If the power does not act at right angles to C B, but obliquely, draw C D perpendicular to the direction of the power, then, by the property of the lever, P : W :: C A : C D.

The capstan, the windlass, and various other contrivances of a similar nature, such as the gimlet and augur for boring holes, &c. may be referred to the same principle. Also, the crank, as used in the steam engine, is a species of wheel and axle. If the force which acts upon a crank presses it directly up and down alternately, the effect, compared with what would take place if the force acted at right angles to the crank all round, is as twice the diameter of a circle to its circumference, or as 2 : 3·14159.

Therefore, to determine the mean length of crank, we must multiply the whole length of the crank by $\frac{2}{3{\cdot}14159}$ or ·6366; that is, when a given force is applied to a given crank as above, to raise a weight, the same effect will be produced if the force be applied at right angles to a crank the length of which is equal to the length of the given crank multiplied by ·6366.

55. When two weights sustain each other by means of a wheel and axle, the thickness of the rope by which each weight is suspended must be taken into the account. We must add half the thickness to the distance at which P and W act respectively. Therefore if R = radius of the wheel, r = radius of the axle, and t = thickness of the rope, then we have $P : W :: r + \frac{1}{2} t : R + \frac{1}{2} t$.

$$\therefore P = \frac{W(r + \frac{1}{2} t)}{R + \frac{1}{2} t}. \qquad (1)$$

$$W = \frac{P(R + \frac{1}{2} t)}{r + \frac{1}{2} t}. \qquad (2)$$

$$R = \frac{W}{P}(r + \tfrac{1}{2} t) - \tfrac{1}{2} t. \qquad (3)$$

$$r = \frac{P}{W}(R + \tfrac{1}{2} t) - \tfrac{1}{2} t. \qquad (4)$$

56. If the wheel be acted on by a power instead of a weight, then the above proportion becomes $P : W :: r + \frac{1}{2} t : R$.

$$\therefore P = \frac{W(r + \frac{1}{2} t)}{R}. \qquad (5)$$

$$W = \frac{P \times R}{r + \frac{1}{2} t}. \qquad (6)$$

$$R = \frac{W(r + \frac{1}{2} t)}{P}. \qquad (7)$$

$$r = \frac{P \times R}{W} - \tfrac{1}{2} t. \qquad (8)$$

57. In the case of two weights P and W, if the thickness of the rope by which P is suspended is not the same as the thickness of the rope by which W is suspended, then, if we put T for the latter thickness, and t for the former, we have

$$P : W :: r + \tfrac{1}{2} T : R + \tfrac{1}{2} t.$$

$$\therefore P = \frac{W(r + \frac{1}{2} T)}{R + \frac{1}{2} t}. \qquad (9)$$

$$W = \frac{P(R + \frac{1}{2} t)}{r + \frac{1}{2} T}. \qquad (10)$$

$$R = \frac{W}{P}(r + \tfrac{1}{2}T) - \tfrac{1}{2}t. \quad (11)$$

$$r = \frac{P}{W}(R + \tfrac{1}{2}t) - \tfrac{1}{2}T. \quad (12)$$

58. By a combination of wheels, we may multiply the power to any extent whatever, by making the lesser wheels to turn the greater.

A combination of wheels and axles is the very same in principle as the combination of levers, given in Art. 41; therefore P : W :: the product of the radii of all the axles : the product of the radii of all the wheels. (See Fig. 26.)

$$\text{Hence } P = \frac{W \times \text{the product of the radii of all the axles}}{\text{the product of the radii of all the wheels}}. \quad (13)$$

$$W = \frac{P \times \text{the product of the radii of all the wheels}}{\text{the product of the radii of all the axles}}. \quad (14)$$

Or, since the number of teeth in wheels are as their radii,

P : W :: the product of the number of teeth in all the pinions : the product of the number of teeth in all the wheels.

$$\therefore P = \frac{W \times \text{the product of the number of teeth in all the pinions}}{\text{the product of the number of teeth in all the wheels}}. \quad (15)$$

$$W = \frac{P \times \text{the product of the number of teeth in all the wheels}}{\text{the product of the number of teeth in all the pinions}}. \quad (16)$$

Example 1.

A weight of 1000 *lbs*. is sustained by a rope of 2 inches diameter, going round an axle the diameter of which is 6 inches; what weight must be suspended at the circumference of the wheel, by a rope of the same thickness, to obtain an equilibrium, the diameter of the wheel being $1\frac{1}{2}$ feet?

By form. 1, we have $P = \dfrac{W(r + \frac{1}{2}t)}{R + \frac{1}{2}t}$

Here $W = 1000$, $r = 3$, $R = 9$, and $t = 2$; therefore $\frac{1}{2}t = 1$.

$$\therefore P = \frac{1000\,(3+1)}{9+1} = \frac{4000}{10} = 400 \text{ lbs.}$$

If the thickness of the rope had not been taken into the account, then,

$$P = \frac{W\,r}{R} = \frac{1000 \times 3}{9} = 333\tfrac{1}{3} \text{ lbs.}$$

Example 2.

A weight of 40 *lbs.* is suspended by a rope of one inch diameter, going round an axle of 4 inches diameter; what power, acting at the circumference of a wheel of 20 inches diameter, will support the equilibrium?

In this case, we must take form. 5, for the power acts upon the wheel without a rope.

$$P = \frac{W\,(r + \frac{1}{2}\,t)}{R} = \frac{40\,(2+\frac{1}{2})}{10} = 10 \text{ lbs.}$$

For here $W = 40$ *lbs.* $r = 2$ inches, $R = 10$ inches; and $t = 1$ inch, or $\frac{1}{2}\,t =$ half an inch.

Example 3.

In a combination of wheels and axles, given the radii of the wheels 24, 26, and 20 inches respectively, and the radii of the pinions 8, 5, and 4 inches; and if a power of 56 *lbs.* be applied to the extremity of the first wheel, what weight will it be able to sustain at the extremity of the last pinion?

By form. 14, $W = \dfrac{56 \times 24 \times 26 \times 20}{8 \times 5 \times 4} = 4368$ *lbs.* $=$ 39 *cwt.*

And, in this case, the power will move 78 times as fast as the weight.

The Pulley.

59. A pulley is a small wheel which turns about an axis passing through its centre. The centre may be either

fixed or moveable. The pulley is either single, or combined together to increase the power.

60. When the power supports the weight by means of a fixed pulley, the power and weight are equal. For, through the centre C draw A B, which will represent a lever of the first order, the fulcrum of which is C, and A C = B C; therefore, since the distance of the power from the fulcrum is equal to the distance of the weight from the fulcrum, to obtain an equilibrium the power must be equal to the weight. (Fig. 27.)

61. When the power sustains the weight by means of a single moveable pulley, the power is but half the weight, if the portions of the sustaining cord are parallel to each other.

For A B may be considered as a lever of the second order, the power acting at A, the weight at C, and the fulcrum or fixed point is at B; therefore P : W :: B C : A B; but B C = $\frac{1}{2}$ A B, consequently P = $\frac{1}{2}$ W. (Fig. 28.)

62. The same principle may be applied to a combination of pulleys, all drawn by one cord going over all the pulleys. (Fig. 29.) Then P : W :: 1 : the number of parts of the cord going round the moveable block.

For the whole weight W is supported by the number of the parts of the cord going round the moveable block; therefore each of these parts must bear an equal portion of the weight; and if n = the number of these parts, then each part must bear the nth part of the weight; or P : W :: 1 : n. The pressure upon the hook A or B = P + W or $(n + 1)$ P.

63. If, instead of the same cord going round all the pulleys, each pulley hangs by a separate cord, then, to obtain an equilibrium, P : W :: 1 : 2^n, n being the number of moveable pulleys. (Fig. 30.)

P : weight at B :: 1 : 2

weight at B : weight at C :: 1 : 2

weight at C : weight at D :: 1 : 2

$$\therefore P : W :: 1 : 2 \times 2 \times 2, \&c. :: 1 : 2^n$$

Cor. Hence $2^n P = W$, or $P = W \div 2^n$.

64. When the strings or cords are not parallel to each other, but form the angle A D E (see Fig. 31), then P : W :: radius : twice the cosine of the angle of inclination of the direction of the power to the direction of the weight.

For produce A B, the direction of the power, to D; and from C, the centre of the moveable pulley, draw C B perpendicular to C D, the direction of the weight; then let D B represent the force in the direction D B, and resolve it into D C, B C, and D C will be that part of it which is effective in sustaining the weight; and since the cord E F sustains the same weight that the cord A B sustains, the whole weight sustained by the cord E F B A will be represented by 2 C D, consequently P : W :: D B : 2 C D :: radius : twice the cosine of the angle B D C.

Example.

If a weight be sustained by a power which is attached to a rope going round a moveable pulley (see Fig. 31), and making an angle of 30° with a vertical line passing through the centre of the pulley, what proportion does the power bear to the weight?

By Art. 64, P : W :: 1 : 2 cos. 30°.

But cos. $30° = \frac{1}{2} \sqrt{3}$.

$\therefore P : W :: 1 : \sqrt{3}$.

If the angle B D C = 45°, then since cos. $45° = \frac{1}{2} \sqrt{2}$, we have $P : W :: 1 : \sqrt{2}$.

If the angle B D C = 60°, its cosine is $\frac{1}{2}$.

$\therefore P : W :: 1 : 1$.

Hence the power and weight are equal.

The Inclined Plane.

65. The Inclined Plane is a plane inclined to the horizon, or a plane which makes any angle whatever with an horizontal plane. (Fig. 32.)

66. If a weight W be sustained upon an inclined plane by a power P, acting in a direction parallel to that plane, then the power P is in proportion to the weight W as the height of the plane is to its length; that is, P : W :: B C : A C.

Draw B D perpendicular to A C. Now the weight W is sustained by three forces, viz. the force of gravity, or its own weight W, acting in the direction B C, perpendicular to the horizon; the power P, acting in the direction D C, parallel to the plane; and the pressure against the plane, acting in the direction D B, perpendicular to the plane. Therefore, by Art. 29, these forces will be respectively proportional to the three sides B C, D C, and D B, of the triangle B D C. But the triangles B D C and A B C are similar, therefore their like sides are proportional; that is, B C, D C, and D B, are to each other respectively as A C, B C, and A B, which are therefore as the three forces W, P, and P′, where P′ represents the pressure against the plane.

That is, the power P is to the weight W :: B C : A C.

And the power P is to the pressure against the plane P′ :: B C : A B.

Also the weight W is to the pressure against the plane P′ :: A C : A B.

$$P = \frac{W \cdot B\,C}{A\,C}. \quad (1)$$

$$W = \frac{P \cdot A\,C}{B\,C}. \quad (2)$$

$$P' = \frac{P \cdot A\,B}{B\,C}. \quad (3)$$

$$P' = \frac{W \cdot A\,B}{A\,C}. \quad (4)$$

Example.

Let the length of the plane A C = 10 feet, the height B C = 6 feet, and the weight 600 *lbs.*; then 10 : 6 :: 600 : 360 *lbs.* the power necessary to sustain the weight; and

$\sqrt{(A\,C^2 - B\,C^2)} = A\,B$, that is $\sqrt{(100 - 36)} = 8 =$ A B, the base of the plane; then 10 : 8 :: 600 *lbs.* : 480 *lbs.* the pressure against the plane.

Or thus, by form. 1, $P = \frac{600 \times 6}{10} = 360$ *lbs.* as before; and by form. 4, $P' = \frac{600 \times 8}{10} = 480$ *lbs.*

67. If the power (Fig. 33), instead of acting in a direction A C, parallel to the plane, should act in a direction D E, making any angle E D C with it, then the power, weight, and pressure against the plane, are respectively as D E, E B, and D B;* for the weight W may be considered as kept in equilibrio by three forces acting in these directions.

Therefore the power P : W :: D E : E B :: sin D B E or sine C A B† : sine E D B.

P : pressure against the plane P′ :: D E : D B :: sine C A B : sine D E B.

Also, the weight W : pressure against the plane P′ :: E B : D B :: sine E D B : sine D E B.

$$\text{Hence the power } P = \frac{W \cdot D\,E}{E\,B} = \frac{W \cdot \text{sine } C\,A\,B}{\text{sine } E\,D\,B}. \quad (1)$$

$$\text{Pressure } P' = \frac{P \cdot D\,B}{D\,E} = \frac{P \cdot \text{sine } D\,E\,B}{\text{sine } C\,A\,B}. \quad (2)$$

$$\text{Also, } P' = \frac{W \cdot D\,B}{E\,B} = \frac{W \cdot \text{sine } D\,E\,B}{\text{sine } E\,D\,B}. \quad (3)$$

Cor. 1. When the power acts in the direction D *e*, parallel to the base of the plane, the three above proportions become

P : W :: D *e* : *e* B :: B C‡ : A B :: sine C A B : cos. C A B.
P : P′ :: D *e* : D B :: B C : A C :: sine C A B : radius.
W : P′ :: *e* B : D B :: A B : A C :: cos. C A B : radius.

* D B is perpendicular to A C, the same as in last article.

† The triangles A C B and D C B are right-angled triangles, and are evidently similar.

‡ The triangles A C B and D *e* B are also similar.

$$P = \frac{W \cdot B\,C}{A\,B} = \frac{W \cdot \text{sine } C\,A\,B}{\text{cos. } C\,A\,B}. \quad (4)$$

$$P' = \frac{P \cdot A\,C}{B\,C} = \frac{P\,.\,\text{radius}}{\text{sine } C\,A\,B}. \quad (5)$$

$$\text{Also } P' = \frac{W \cdot A\,C}{A\,B} = \frac{W \cdot \text{radius}}{\text{cos. } C\,A\,B}. \quad (6)$$

Cor. 2. The least power will be necessary to sustain a given weight when it acts in a direction parallel to the plane; for, by form. 1, $P = \frac{W \cdot \text{sine } C\,A\,B}{\text{sine } E\,D\,B}$, and since W and sine C A B are both given, therefore P is proportional to $\frac{1}{\text{sine } E\,D\,B}$, and will evidently be the least possible when sine E D B is the greatest; that is, when E D B is a right angle, or E D coincides with C D.

Cor. 3. The pressure against the plane will be greatest when the power acts in a direction parallel to the base of the plane; for, by form. 2, $P' = \frac{P \cdot \text{sine } D\,E\,B}{\text{sine } C\,A\,B}$, and supposing P and the sine C A B given, then P′ is proportional to sine D E B, and is therefore greatest when sine D E B is the greatest; that is, when D E B is a right angle, or when D E coincides with D *e*.

Example 1.

If a weight of 150 *lbs.* be sustained on an inclined plane by a power acting in a direction parallel to the base of the plane, the length of the plane being 10 feet, and the base 8 feet, required the pressure against the plane.

By Art. 67, Cor. 1, form. 6, we have $P' = \frac{W \times A\,C}{A\,B}$.

Here W = 150, A C = 10 feet, A B = 8 feet; hence $P' = \frac{150 \times 10}{8} = 187\frac{1}{2}$ *lbs.*

Example 2.

Compare the pressures against an inclined plane in the two following cases:

1st, When a body is sustained on an inclined plane by a power acting parallel to the plane.

2d, When a body is sustained on an inclined plane by a power acting parallel to the base of the plane.

Let p represent the pressure against the plane in Case 1st, and P′ that in Case 2d. Then, by Art. 66,

$$W : p :: A\,C : A\,B.$$

And by Art. 67, Cor. 1, we have

$$P' : W :: A\,C : A\,B.$$

$$\text{ex æquo } P' : p :: A\,C^2 : A\,B^2.$$

That is, the pressure in the latter case : the pressure in the former :: the square of the length of the plane : the square of the base of the plane.

Let the inclination of the plane be 30°, then the length is to the base as $1 : \frac{1}{2}\sqrt{3}$; hence the above pressures are in proportion to each other as $1 : \frac{3}{4}$, or as 4 : 3; and if the inclination of the plane is 60°, the length of the plane is double the base; therefore the above pressures are, in this case, as $2^2 : 1^2$, or in proportion to each other as 4 : 1.

Example 3.

Two inclined planes A B and B C have the same height B D, and upon these planes two weights P and W keep each other in equilibrio by means of a string going over a pulley fixed at B, the parts of the string B W and B P being parallel to the planes. Prove that P : W :: B C : A B. (Fig. 34.)

By form. 1, the power necessary to sustain W on the plane $A\,B = \dfrac{W \times B\,D}{A\,B}$, and in the same manner the power necessary to sustain P on the plane $B\,C = \dfrac{P \times B\,D}{B\,C}$

Now whatever power is exerted at B to sustain W on the

plane A B, the very same power must be exerted at that point to sustain P upon the plane B C in the case of an equilibrium; therefore $\frac{W \times B\,D}{A\,B} = \frac{P \times B\,D}{B\,C}$.

$\therefore W \times B\,D \times B\,C = P \times B\,D \times A\,B$, or $W \times B\,C = P \times A\,B$. Hence, $P : W :: B\,C : A\,B$.

The Wedge.

68. The Wedge is an instrument made of iron or some hard substance. Its form, in the most useful cases, is that of a prism contained between two isosceles triangles, as A C B. (Fig. 35.)

69. In the wedge A C B, if the power acting perpendicularly to the back A B is to the force acting perpendicularly against either side A C or B C, as the breadth of the back A B is to the length of the side A C or B C, the wedge will be in equilibrio.

For, by Art. 29, Cor. 2, when three forces are in equilibrium, they are as the corresponding sides of a triangle drawn perpendicular to the directions in which these forces act. But A B is perpendicular to the direction of the force against the back, and A C, B C are perpendicular to the forces acting against them; therefore the three forces are as A B, A C, B C.

Cor. If we take into the account the resistance at both sides of the wedge, then, if there is an equilibrium, the power at D is to the whole resistance as the back A B is to the sum of the sides A C, B C, or as A B to 2 A C or 2 B C.

In general, the wedge is used for splitting or cleaving wood, and separating the parts of hard bodies, by a blow from a hammer or mallet. The force impressed by percussion, or a blow on the back of the wedge, produces an effect incomparably greater than any dead weight or pressure, such as is employed in the other mechanical powers. And it may also be observed, that the wedge is seldom

urged otherwise than by percussion; and very little can be gathered from the theory, but that the thinner the wedge is, the greater is its power.

The Screw.

70. The Screw (Fig. 36) is a spiral groove or thread, winding round a cylinder, so as to cut all the lines drawn on its surface parallel to its axis at the same angle. The spiral may be either on the convex or concave surface of the cylinder, and it is called accordingly either the screw or the nut.*

71. If the power be applied parallel to the base of the screw, and perpendicular to the radius of the cylinder, and if the weight press perpendicularly on the axis, an equilibrium is produced when the power is to the weight or resistance, as the distance between two threads of the screw to the circumference described by the point to which the power is applied.

For the screw is nothing more than the inclined plane A B C wrapped round the cylinder, the base A B of the plane being equal to the circumference of the cylinder's base, and coinciding with it, and the height B C of the plane equal to the distance between two of the threads; and since the power in this case acts parallel to the base, we have, by Art. 67, Cor. 1, P : W :: B C : A B :: the distance between two of the threads : the circumference described by the power, which, in this case, is the circumference of the cylinder.

But when the power is applied at any other point, as P, the effect of that power will be increased in the proportion of O P to O A, or as the circumference described by the point P to the circumference described by the point A; therefore P : W :: the distance between two of the threads

* Or they are sometimes called the exterior and interior screw.

: the circumference described by the power, let this circumference be what it may.

Example 1.

Given the distance O P at which the power acts = 3 feet, and the distance between two of the threads 2 inches, to find what weight a man would be able to sustain when he acts at P with a force of 150 *lbs.*

Now 6 feet = 72 inches, and $72 \times 3{\cdot}1416 = 226{\cdot}1952$ inches = the circumference described by the power, therefore, by Art. 71,

$$P : W :: 2 : 226{\cdot}1952.$$

But P = 150 *lbs.* hence $W = \frac{226{\cdot}1952 \times 150}{2} =$ 16964·64 *lbs.*

When the screw is applied as in Fig. 37, it is called an endless screw; we have here also a combination of wheels, and, to obtain an equilibrium, we must have recourse to Art. 58, from which we have,

$P \times 2\,A\,C \times 3{\cdot}1416 \times$ radii of all the wheels $= W \times$ distance of two threads $\times$ radii of all the pinions.

Example 2.

If the endless screw A B (Fig. 37) be turned by a handle A C of 20 inches, the threads of the screw being distant half an inch each; and the screw turns a toothed wheel E, the pinion L of which turns another wheel F, and the pinion of this another wheel G, to the pinion or barrel of which is hung a weight W; it is required to determine what weight a man will be able to sustain who acts at the handle C D with a force of 150 *lbs.* supposing the diameters of the wheels to be 18 inches, and those of the pinions and barrel 2 inches.

From what has been premised, we have $150 \times 40 \times 3{\cdot}1416 \times 18^3 = W \times \frac{1}{2} \times 2^3$.

Hence $W = \frac{150 \times 40 \times 3{\cdot}1416 \times 18^3}{\frac{1}{2} \times 2^3} = \frac{109930867{\cdot}2}{4}$

= 27482716·8 *lbs.* or 12269 tons.

ON THE CENTRE OF GRAVITY.

72. The centre of gravity of any body or system of bodies is that point about which the body or system, acted upon only by the force of gravity, will balance itself in any position whatever: or it is that point, which being supported, the body or system will be supported, however it may be situated; and that centre will always tend to descend to the lowest place to which it can get, when it is not the point of suspension.

73. The centre of gravity of a body is not always within the body itself. Thus, the centre of gravity of a ring is not in the substance of the ring, but in the centre of its circumscribing circle; and the centre of gravity of a hollow staff or bone is somewhere in its imaginary axis.

74. If a line drawn perpendicular to the horizon, from the centre of gravity, fall within the base of any body, it will rest in that position; but if the perpendicular fall without the base, the body will fall down.

For when the perpendicular falls within the base, the body can be moved in no manner whatever but the centre of gravity will rise. And if the perpendicular fall without the base towards any side, the body cannot be moved towards that side but the centre of gravity descends, and therefore the body will fall that way.

Cor. 1. If a perpendicular, drawn from the centre of gravity, fall just on the extremity of the base, the body may stand; but the least force whatever will cause it to fall that way: and the nearer the perpendicular is to any side, or the narrower the base is, the easier it will be made to fall, or be pushed over that way.

Cor. 2. Hence, if the centre of gravity of a body be supported, the whole body is supported; and the place of

the centre of gravity may, in many inquiries, be accounted the place of the body; for we may consider the whole mass or quantity of matter to be concentrated in that point, and therefore all the force also with which it endeavours to descend.

Cor. 3. If a body be laid upon a plane G B (Fig. 38), and one end A be gradually raised up, the body will slide down the plane if the perpendicular C D fall within the base; but if the perpendicular fall without the base, the body will roll down the plane.

It may be remarked here, that an equilibrium may take place when the centre of gravity is at the highest point to which it can ascend; but then this is only a tottering equilibrium, which the least motion will destroy; and the body or system, after the equilibrium is destroyed, will vibrate till the centre of gravity has obtained the lowest place of its descent.

75. The common centre of gravity C of any two bodies A, B, divides the line joining their centres into two parts, which are reciprocally as the bodies. A C : B C :: B : A. (Fig. 39.)

For if the centre of gravity C be supported, the two bodies A and B will be supported, and will rest in equilibrio. But by the property of the lever, when two bodies are in equilibrio about C as a fulcrum, we have $A \times A\,C = B \times B\,C$, or A C : B C :: B : A.

Cor. Hence, by composition, $A + B : B :: A\,B : A\,C$.

To find the centre of gravity of any number of bodies connected together by inflexible right lines without weight (Fig. 40).

76. Let A, B, C, D, E, represent the bodies, the centre of gravity of which is required. Join any two of them, as A and B, by the right line A B; which divide in E, so that $A + B : B :: A\,B : A\,E$; and by Cor. to the last Art. E will be the centre of gravity of A and B. We must now suppose the sum of the bodies A and B to be collected in

E, and join E, C, by the right line E C, and divide it in F, so that A + B + C : C :: E C : E F; then will F be the centre of gravity of A, B, C.

Lastly, join F, D, and suppose the sum of the bodies A, B, C, to be collected in F; then divide F D in G, so that A + B + C + D : D :: F D : F G, and G will be the centre of gravity of the four bodies A, B, C, D. And in the very same manner, the centre of gravity of any number of bodies may be found.

77. Let A, B, be any two bodies, and C their centre of gravity. If any point P be taken in the line A B, then $PA \times A + PB \times B = PC. (A + B)$, if the bodies are on the same side of the point P; and $PA \times A - PB \times B = PC. (A + B)$, when they are on opposites sides of that point. (Fig. 41.)

For, by Art. 75, $A \times AC = B \times BC$; and when the bodies are both on the same side of the point P, we have $AC = PA - PC$, and $BC = PC - PB$; therefore $A. (PA - PC) = B. (PC - PB)$. Or $PA \times A - PC \times A = PC \times B - PB \times B$, transpose and $PA \times A + PB \times B = PC \times A + PC \times B = PC. (A + B)$.

But if they are on contrary sides, then $AC = PA - PC$, and $BC = PB + PC$; therefore $A. (PA - PC) = B. (BP + PC)$, or $PA \times A - PC \times A = PB \times B + PC \times B$, and by transposition $PA \times A - PB \times B = PC \times A + PC \times B = PC. (A + B)$.

Cor. 1. Hence the bodies A and B have the same force to turn the lever A P about the point P, as if they were both placed at C, their centre of gravity.

Cor. 2. In a manner similar to the above, it may be shown that whatever may be the number of bodies A, B, D, &c. we have $PA \times A + PB \times B + PD \times D$, &c. $=$ $PC. (A + B + D$, &c.), when the bodies are all on the same side of the point P; but if on contrary sides, then the difference of the products must be taken.

To find the centre of gravity of the area of a parallelogram (Fig. 42).

78. Bisect A B in E, and A D in G; draw E F parallel to A D, and G H parallel to A B; their intersection O is the centre of gravity required. For E F bisects every right line that can be drawn parallel to A B; therefore the centre of gravity must be somewhere in E F. G H also bisects every right line that can be drawn parallel to A D; therefore the centre of gravity must be somewhere in the line G H. But it can only be in both these lines when it is at O, their point of intersection; therefore O is the centre of gravity of the area of the parallelogram A B C D.

To find the centre of gravity of a triangle (Fig. 43).

79. Let A B C be any triangle, and from any two of its angles B and C draw the right lines C D and B E to bisect the opposite sides in D and E; then will their intersection G be the centre of gravity of the triangle.

For since C D bisects A B, it will bisect all the right lines that can be drawn parallel to A B; that is, all the parallel sections of the figure; therefore the centre of gravity of the triangle lies in C D. For the same reason, it also lies in B E; consequently it is in G, their common point of intersection.

Cor. Join D E, which will be parallel to B C, and equal to one half of it, and the triangles B G C and E G D are similar; therefore since D E $= \frac{1}{2}$ B C, D G $= \frac{1}{2}$ D C, or C G $= \frac{2}{3}$ C D. In the same manner, B G $= \frac{2}{3}$ B E.

To find the centre of gravity of a trapezium (Fig. 44).

80. Divide the trapezium A B C D into two triangles by the diagonal B D, and find the centres of gravity E and F of these two triangles; then the centre of gravity of the trapezium will lie in the line E F, joining them. If we suppose the weight of each triangle to be collected in E

and F, then by Cor. to Art. 75, if E F be divided in G, so that E + F : F :: E F : E G (that is, the area of the trapezium A B C D : area of the triangle B C D :: E F : E G), then G is the centre of gravity required.

Or, having found the centres of gravity E, F, if the trapezium be divided into two other triangles B A C, D A C, by the other diagonal A C, and the centres of gravity H and I of these two triangles be found, then the centre of gravity of the trapezium will lie in the line H I.

But it has also been proved that the centre of gravity of the trapezium lies in E F; therefore, since it lies both in E F and H I, it must necessarily lie in G, their common point of intersection.

In the same manner, we may find the centre of gravity of a figure of any number of sides. For it may be divided into triangles, the centres of gravity of which may be separately found; after which, we can find the centre of gravity of the whole of the triangles, by considering the whole weight of each to be collected in its centre of gravity.

1. The centre of gravity of a cylinder, or any other body the parallel sections of which are equal, is in the middle of the axis of that body.

2. For the arc of a circle, as $\frac{1}{2}$ arc : sine of $\frac{1}{2}$ arc :: radius : distance of its centre of gravity from the centre.

3. For the sector of a circle, as arc : chord :: $\frac{2}{3}$ radius : distance of its centre of gravity from the centre.

4. For a parabolic space, the distance of the centre of gravity from the vertex is $\frac{3}{5}$ of the axis.

5. The centres of gravity of the surface of a cylinder, of a cone, and of a conic frustrum, are respectively at the same distances from the origin as are the centres of gravity of the parallelogram, triangle, and trapezoid, which are vertical sections of the respective solids.

6. In a cone, as well as any other pyramid, the distance of the centre of gravity from the vertex is $\frac{3}{4}$ of the axis.

7. In a conic frustrum, the distance on the axis from the centre of the less end is $\frac{1}{4} h. \frac{3 R^2 + 2 R r + r^2}{R^2 + R r + r^2}$, where h denotes the height, and R and r the radius of the greater and lesser ends.

8. The same theorem will serve for the frustrum of any regular pyramid, taking R and r for the sides of the two ends.

9. In the paraboloid, the distance from the vertex is $\frac{2}{3}$ of the axis.

10. In the frustrum of the paraboloid, the distance on the axis from the centre of the less end is $\frac{1}{3} h. \frac{2 R^2 + r^2}{R^2 + r^2}$ where h denotes the height, R and r the radii of the greater and lesser ends.*

Example 1.

Given the weights of two bodies 50 and 20 *lbs.* and distance asunder 35 feet; how far from the larger body is their common centre of gravity?

By Art. 75, Cor. 50 + 20 : 20 :: 35 : 10 feet.

Therefore the centre of gravity is 10 feet from the larger body, and 25 feet from the smaller body.

Example 2.

If three equal bodies be placed at the angles of any triangle, prove that the common centre of gravity of these bodies is in the same point with the centre of gravity of the triangle.

The bodies A, B, and C (Fig. 45), being all equal, the centre of gravity of any two of them, suppose A and B, will be in D, the middle of the side A B. The bodies A and B must now be supposed to be both collected in D, and join D, C, by the right line D C; and since A + B

* The above are collected from the Mechanics of Emerson and Dr. Gregory.

$= 2$ C, then, by Cor. to Art. 75, C G : D G :: 2 C : C, or C G : D G :: 2 : 1. Hence C G $= 2$ D G, or C G $= \frac{2}{3}$ C D. Therefore, by Cor to Art. 79, G is the centre of gravity of the triangle A B C.

Example 3.

If five bodies, the weights of which are 3, 6, 9, 5, and 4 *lbs.* are placed at the distance of 2, 3, 4, 5, and 7 feet, respectively, from a given point, and on the same side of it; what is the distance of their common centre of gravity from the given point?

By Art. 77, Cor. 2, we have $2 \times 3 + 3 \times 6 + 4 \times 9 + 5 \times 5 + 7 \times 4 = (3 + 6 + 9 + 5 + 4)$ multiplied by the distance of their common centre of gravity from the given point. Consequently, that distance is equal to

$$\frac{2 \times 3 + 3 \times 6 + 4 \times 9 + 5 \times 5 + 7 \times 4}{3 + 6 + 9 + 5 + 4} = \frac{113}{27} = 4\tfrac{5}{27}$$

feet, the distance required.

Example 4.

If a homogeneous beam be 16 feet long, each foot weighing 5 *lbs.* and a weight of 60 *lbs.* is suspended at one end, what point of the beam will be the centre of gravity?

We must consider the whole weight of the beam to be collected in its centre of gravity, which will be in its middle point, as the beam is uniform.

Then $16 \times 5 = 80$ *lbs.* the whole weight of the beam.

Then, by Art. 75, Cor. $80 + 60 : 60 :: 8 : 3\frac{3}{7}$, which is the distance between the centre of gravity and the middle of the beam; consequently, $8 + 3\frac{3}{7} = 11\frac{3}{7}$, the distance between the centre of gravity and the end.

And $16 - 11\frac{3}{7} = 4\frac{4}{7}$, the distance between the centre of gravity and the end where the weight is suspended.

Or it may be found thus: $80 + 60 : 80 :: 8 : 4\frac{4}{7}$.

Hence we have the following rule:

Multiply the whole weight of the beam by half its length for a dividend.

Then add the whole weight of the beam and the weight suspended at the end together for a divisor.

And if the above dividend be divided by this divisor, the quotient will give the distance of the centre of gravity from the end where the weight is suspended.

Example 5.

If the height of a cylinder be double the diameter of its base, what is the angle of inclination of its axis with the horizon when it is just ready to fall over? (Fig. 46.)

By Art. 74, Cor. 1, the inclination of the axis of the cylinder must be such that a perpendicular, drawn from the centre of gravity G of the cylinder, will fall just on the extremity A of the base. Produce the axis of the cylinder to B; then the triangles G A B, A E B, and A E G, are all right-angled and similar, by Euclid, Book 6, Prop. 8; and since the altitude of the cylinder is double the diameter of its base, G E $=$ 2 A E, consequently the angle G A E $=$ twice the angle A G E, whence the angle G A E $= 60^\circ$, and the angle A G E $= 30^\circ$; but the angle G A E is equal to the angle A B E, which is the angle of inclination of the axis of the cylinder with the horizon; therefore the axis makes an angle A B E of 60° with the horizon.

Also, since the angle A G E is equal to the angle B A E, the base of the cylinder will make an angle of 30° with the horizon.

If the cylinder be oblique (Fig. 47), then C A will be equal to the altitude of the cylinder; but C A is equal to twice H A, therefore G A is equal to twice B A; hence the angle G B A is equal to twice the angle B G A. Therefore the angle G B A, which is the angle of inclination of the axis of the cylinder with the horizon, is 60°, the same as above.

EXAMPLES FOR PRACTICE.

Example 1.

A weight of $1\frac{1}{2}$ *lbs.* laid on the shoulder of a man, is no greater burden to him than its absolute weight, or 24 ounces; what difference will he feel between the said weight applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand, 28 inches therefrom; and how much more must his muscles then draw to support it at right angles, that is, having his arm extended right out?

$$\text{Now } 1{\cdot}5 \textit{ lbs.} \times 12 \text{ inches} = 18 \textit{ lbs.}$$
$$\text{and } 1{\cdot}5 \textit{ lbs.} \times 28 \text{ inches} = 42 \textit{ lbs.}$$
$$\text{Then } 42 \textit{ lbs.} - 18 \textit{ lbs.} = 24 \textit{ lbs.}$$

Example 2.

If a weight W be sustained on a horizontal plane by three props which are not in the same straight line, the pressure on each will be the same as if a single weight were laid on it, so that the sum of all the three weights were equal to W, and their common centre of gravity the same with the centre of gravity of that body.

If the props be A, B, C, (Fig. 48) and if W be placed with its centre of gravity at D, and if A D G, B D F, C D E, be drawn, then the pressure

$$\text{on A} = \frac{DG}{AG} \times W,$$

$$\text{on B} = \frac{DF}{BF} \times W,$$

$$\text{on C} = \frac{DE}{CE} \times W,$$

When D coincides with the centre of gravity of the triangle A B C, the pressure on each of the props is the

same; for, in this case, D G is one-third of A G, D F is one-third of B F, and D E is one-third of C E; therefore each prop sustains one-third of the weight.

If the weight be supported on more than two props, the problem appears to admit of innumerable solutions; but if the centre of gravity of W, as it rests on the plane, be the same with the centre of gravity of the figure made by joining the tops of the props by straight lines, the pressures on the props are all equal to one another.

Example 3.

A uniform beam of timber, 10 feet in length, being suspended at the distance of 8 feet from one of the ends, it required a weight of 2 *cwt.* attached to the other end to keep it parallel to the horizon; what is the weight of the beam?

Rule.—Multiply the weight which is suspended at the end by its distance from the fulcrum, and take twice this product for a dividend. Then subtract the square of the distance between the fulcrum and the end where the weight is suspended, from the square of the distance between the fulcrum and the other end, for a divisor. And if the above dividend be divided by this divisor, the quotient will give the weight of one foot or one inch in length, according as you take the length in feet or inches; and this quotient, multiplied by the whole length of the beam, will give the whole weight of the beam.

Here 2 feet is the distance between the fulcrum and the end where the weight is suspended, and 8 feet is the distance between the fulcrum and the other end.

By the rule, we have $\frac{2 \times 2 \times 224}{8^2 - 2^2} = \frac{896}{60} = 14\frac{14}{15}$ *lbs.* the weight of one foot in length of the beam; therefore $14\frac{14}{15} \times 10 = 149\frac{1}{3}$ *lbs.* the whole weight of the beam.

Example 4.

Two men carrying a burden of 200 *lbs.* weight between them, hung on a pole, the ends of which rest on their shoulders; how much of this load is borne by each man, the weight hanging 6 inches from the middle, and the whole length of the pole being 4 feet?

Now 4 feet = 48 inches, and the burden being hung on at 6 inches from the middle, it will be 18 inches from one of the men, and 30 inches from the other.

And by the note at page 17, we have 48 : 200 :: 30 : 125 *lbs.* the weight borne by the man who is nearest to the burden.

And 48 : 200 :: 18 : 75 *lbs.* the weight borne by the other man.

Example 5.

If the altitude of a cone be double the diameter of its base, what is the inclination of its axis with the horizon when it is just ready to fall over? (Fig. 49.)

By Art. 74, Cor. 1, when the cone is just ready to fall over, a perpendicular from the centre of gravity will fall on the extremity A of the base.

From the centre of gravity of a cone (page 46) we have B D = 4 G D, and by the question B D = 2 A C = 4 A D; hence G D = A D, and therefore the angles G A D and A G D are each 45°, or half a right angle. But the triangles A E G, G A D, and D A E, are all similar (Euclid, Book 6, Prop. 8); consequently, the angle A E G, which is the angle of inclination of the axis with the horizon, is 45°; and its complement, the angle D A E, which is the angle the base of the cone makes with the horizon, is also 45°.

Example 6.

Two inclined planes A B and B C have the same height, and upon these planes two weights keep each other in

equilibrio in the same manner as in Example 3, page 38: given the length of the planes 30 and 40 inches respectively, and the horizontal distance A C between the feet of the planes 50 inches; required their common height, and the ratio of the weights.

Rule.—As the base or longest side is to the sum of the other two sides, so is their difference to the difference of the segments of the base. And half the difference of the segments, added to half their sum, gives the greater segment; and half the difference of the segments, subtracted from half their sum, will give the lesser segment.

Thus, $50 : 40 + 30 = 70 :: 40 - 30 = 10 : 14$, the difference of the segments of the base; and $\frac{14}{2} = 7$, half the difference of the segments.

Therefore $25 + 7 = 32$, the greater segment; hence 18 is the lesser segment.

Then $\sqrt{40^2 - 32^2} = 24$, the common height of the planes.

And, by Example 3, pages 38 and 39, P : W :: B C : A B.

Here B C $= 40$, and A B $= 30$; therefore P : W :: $40 : 30 :: 4 : 3$. That is, the weights must be in proportion to each other as 4 to 3.

Example 7.

A cast iron beam, of uniform thickness, and in the form of a parabola, is supported upon two pins, one of which is fixed at the vertex, and the other at the middle of the base: required the pressure on each pin when the distance between the vertex and the middle of the base is 5 feet, the base 2 feet, and the weight of the beam 56 *lbs*.

Here, in this Example, the centre of gravity is not in the middle of the beam; and by Form. 4, page 46, the distance of the centre of gravity from the vertex of a parabola is $\frac{3}{5}$ of the axis.

Hence $\frac{3}{5} \times 5 = 3$ feet, the distance of the centre of gravity from the vertex.

Now we must consider the whole weight of the beam to be collected in its centre of gravity; and by note, page 17, we have $5 : 56 :: 3 : 33\frac{3}{5}$ *lbs.* the pressure on the pin which is fixed in the middle of the base.

And $5 : 56 :: 2 : 22\frac{2}{5}$ *lbs.* the pressure on the pin which is fixed in the vertex.

STRENGTH AND STRESS OF MATERIALS.

81. A knowledge of the strength and stress of materials is of so much importance to the practical mechanic, that it demands his most serious attention; and he will find the works of Barlow and Tredgold* to be treasures of inestimable value, being fraught with every kind of useful information relating to these subjects.

Mr. Barlow shows that there are four distinct strains to which every hard body may be exposed, and which may be stated as follow:—

1. A body may be pulled or torn asunder by a stretching force, applied in the direction of its fibres; as in the case of ropes, stretchers, king-posts, tie-beams, &c.

2. It may be broken across by a transverse strain, or by a force acting either perpendicularly or obliquely to its length; as in the case of levers, joists, &c.

* Barlow on the Strength and Stress of Timber, and Tredgold on the Strength of Cast Iron.

3. It may be crushed by a force acting in the direction of its length; as in the case of pillars, posts, and truss-beams.

4. It may be twisted or wrenched by a force acting in a circular direction; as in the case of the axle of a wheel, the nail of a press, &c.

On the Cohesive Strength of Materials.

82. The force of cohesion may be defined to be that force by which the fibres or particles of a body resist separation, and is therefore proportional to the number of fibres in the body, or to the area of its section.

Mr. Emerson gives the load that may be safely borne by a square inch rod of each of the following:—

	Pounds Avoirdupois.
Iron rod, an inch square, will bear ...	76,400
Brass, ...	35,600
Hempen rope, ...	19,600
Ivory, ...	15,700
Oak, Box, Yew, Plum-tree, ...	7,850
Elm, Ash, Beech, ...	6,070
Walnut, Plum, ...	5,360
Red fir, Holly, Elder, Plane, Crab, ...	5,000
Cherry, Hazel, ...	4,760
Alder, Asp, Birch, Willow, ...	4,290
Lead, ...	430
Freestone, ...	914

He also gives the following practical rule, viz. That a cylinder, the diameter of which is d inches, loaded to one-fourth of its absolute strength, will carry as follows:—

	Cwt.
Iron, ...	$135 \times d^2$
Good rope, ...	$22 \times d^2$
Oak, ...	$14 \times d^2$
Fir, ...	$9 \times d^2$

Also he says that a cylindric rod of good clean fir, of an inch circumference, drawn in length, will bear at its extremity 400 *lbs.*; and a spear of fir, 2 inches diameter, will bear about 7 tons, but not more.

A rod of good iron, of an inch circumference, will bear near 3 tons weight.

A good hempen rope, of an inch circumference, will bear 1000 *lbs.* being at its extremity.

The following interesting experiment was made at the Patent Iron Cable Manufactory of Capt. S. Brown (see Barlow's Essay, 2d Edition, p. 257):—

A bolt of Welsh iron, 12 feet 6 inches long, and 2 inches in diameter, required a strain of 82 *tons* 15 *cwt.* to tear it asunder. When subject to a strain of 68 *tons*, it stretched 3 inches, and was reduced to $1\frac{15}{16}$ inch in diameter. When the strain was increased to 74 *tons* 15 *cwt.* it had stretched 6 inches, and was reduced $\frac{1}{8}$ of an inch gradually in the diameter. With 82 *tons*, it stretched 14 inches. With 82 *tons* 15 *cwt.* the bolt broke about 5 feet from the end, the levers being exactly balanced. It had stretched during the whole process $18\frac{1}{2}$ inches, and measured at the place of rupture $1\frac{5}{8}$ inch in diameter.

A bar of cast iron, Welsh pig, $1\frac{1}{4}$ inch square, 3 feet 6 inches long, required a strain of 11 *tons* 7 *cwt.* to tear it asunder; broke exactly transverse, without being reduced in any part; quite cold when broken; particles fine, dark blueish grey colour.

Mr. Barlow gives the mean of Capt. Brown's experiments	25 *tons*.
The mean of Mr. Telford's experiments	$29\frac{1}{4}$ *tons*.
And the mean of the two is ...	27 *tons* nearly,

which may be safely assumed as the medium strength of an iron bar 1 inch square. (Barlow's Essay, 2d edition, page 258.)

Mr. Barlow gives the following table, as a mean derived from his experiments, on the strength of direct cohesion on a square inch of the following:—

	lbs.
Box is about	20,000
Ash, ...	17,000
Teak, ...	15,000
Fir, ...	12,000
Beech, ...	11,500
Oak, ...	10,000
Pear, ...	9,800
Mahogany, ...	8,000

He also gives the following practical rule, for finding the cohesive strength:—

Multiply the number of square inches in the section of any of the bodies in the above table, by its corresponding tabular number, and this product will give the strength required.

83. Mr. Barlow observes, that the above table gives the absolute and ultimate strength of the fibres; and therefore, if the quantity that may be safely borne be required, not more than two-thirds of the above values must be used, or perhaps not more than one-half. He says that, in many of his experiments, he has left more than three-fourths of the whole weight hanging for 24, and even 48 hours, without perceiving the least change in the state of the fibres, or any diminution of their ultimate strength.

On the Transverse Strength of Beams, &c.

84. The transverse strength of rectangular beams, or the resistance which they offer to fracture, is as the breadth and square of the depth: therefore, if two rectangular beams have the same depth, their strengths are to each other as their breadths; but if their breadths are the

same, then their strengths are to each other as the squares of their depths.

85. The transverse strengths of square beams are as the cubes of the breadths or depths. Also, in cylindrical beams, the transverse strengths are as the cubes of the diameters.

Thus, if a beam which is one foot broad and one foot deep support a given weight, then a beam of the same depth and 2 feet broad will support double the weight.

But if a beam be one foot broad and 2 feet deep, it will support four times as much as a beam one foot broad, and one foot deep.

If a beam one foot square support a given weight, then a beam 2 feet square will support eight times as much. Also, a cylinder of 2 inches in diameter will support eight times as much as a cylinder one inch in diameter.

The following table of data is extracted from tables in Barlow's Essay:—

Teak, ...	2462
English Oak, ...	1672
Canadian do.	1766
Dantzic do.	1457
Adriatic do.	1388
Ash, ...	2026
Beech, ...	1556
Elm, ...	1013
Pitch Pine, ...	1632
Red Pine, ...	1341
New England Fir, ...	1102
Riga Fir, ...	1108
Mar Forest Fir, ...	1262
Larch, ...	1127

Case 1.

To find the ultimate transverse strength of any rectangular beam of timber, fixed at one end, and loaded at the other.

Rule 1. Multiply the value given in the table of data by the breadth and square of the depth, both in inches, and divide that product by the length, also in inches, and the quotient will be the weight in pounds.

Example 1.

What weight will it require to break a piece of Mar Forest fir, the breadth being 3 inches, depth 4 inches, and length 4 feet?

In the table of data, opposite Mar Forest fir, stands 1262.

Then, by the rule, $\frac{1262 \times 3 \times 4^2}{48} = 1262$ *lbs.*

Example 2.

What weight will it require to break a beam of English oak, the breadth of which is 2 inches, depth 6 inches, and length 12 feet?

Opposite English oak, in the table, stands 1672.

Then $\frac{1672 \times 2 \times 6^2}{144} = 836$ *lbs.*

Or, if the dimensions of a beam be required, so as to support a given weight at its end, then—

Multiply the weight in pounds by the length in inches; and this product, divided by the tabular value, will give the product of the breadth and square of the depth.

Example 1.

Required the dimensions of a beam of larch, 10 feet long, so as to be capable of supporting a weight of 1000 *lbs.* at one end, the other end being fixed in a wall.

The tabular value for larch is 1127.

Therefore $\frac{120 \times 1000}{1127} = 106{\cdot}5$ nearly, $=$ the breadth and square of the depth.

Let the breadth be 2 inches, then $\frac{106{\cdot}5}{2} = 53{\cdot}25$, the square of the depth; and $\sqrt{53{\cdot}25} = 7{\cdot}3$ inches the depth.

Example 2.

A square balk of ash projects 4 feet 6 inches from a solid wall in which it is fixed; what must be the side of its square, so that the balk may be able to support 1013 *lbs.*?

The tabular value for ash is 2026.

$\frac{54 \times 1013}{2026} = \frac{54702}{2026} = 27$, the cube root of which is 3 inches, the side of the square required.

Case 2.

To compute the ultimate transverse strength of any rectangular beam, when supported at both ends, and loaded in the centre.

Rule. Multiply the value given in the table of data by four times the breadth and square of the depth in inches, and divide that product by the length, also in inches, for the weight.

Example 1.

What weight will be necessary to break a beam of Canadian oak, the length being 10 feet, the breadth 6 inches, and the depth 10 inches; being supported at each end, and loaded in the middle?

For Canadian oak the tabular value is 1766.

$\frac{1766 \times 4 \times 6 \times 10^2}{120} = \frac{4238400}{120} = 35320$ *lbs.**

* If the beam be supported at both ends, and loaded uniformly throughout its length, the result must be doubled.

Example 2.

What weight will it require to break a beam of Mar Forest fir, which is 5 inches broad, 6 inches deep, and 20 feet between the supports?

The tabular value for Mar Forest fir is 1262.

By the rule, $\frac{1262 \times 4 \times 5 \times 6^2}{240} = \frac{1262 \times 20 \times 36}{240} =$ 3786 *lbs.*

If the dimensions of a beam be required, so as to support a given weight, the following rule must be used.

Rule.—Multiply the weight in pounds by the length in inches; and this product, divided by the tabular value, will give the product of four times the breadth and square of the depth: therefore, the breadth being known, we can find the depth; or, the depth being known, we can find the breadth.

Example.

What must be the dimensions of a beam of Dantzic oak, 20 feet long between the supports, to bear a weight of 2 tons in the middle of its length?

The tabular value for Dantzic oak is 1457.

$\frac{4480 \times 240}{1457} = 670$ nearly, which is the square of the depth multiplied by 4 times the breadth.

Let the breadth be 5 inches; then $5 \times 4 = 20 = 4$ times the breadth.

Then $\frac{670}{20} = 33{\cdot}5 =$ the square of the depth;

and $\sqrt{33{\cdot}5} = 5{\cdot}34$ nearly, the depth.

If the beam be fixed at both ends, and loaded in the middle, the result must be increased by its half.

If the beam be fixed at both ends, and loaded uniformly throughout its length, the result must be multiplied by 3.

Or, if we take the depth 4 inches, then $4^2 = 16$, and $\frac{670}{16} = 41{\cdot}875 = 4$ times the breadth.

Therefore, $\frac{41{\cdot}875}{4} = 10{\cdot}46875 =$ breadth.

On the Transverse Strength of Cast Iron.

Case 1.

Rule 1.—To find the breadth of an uniform cast iron beam, to bear a given weight in the middle.

Multiply the length of the bearing in feet, by the weight to be supported in pounds, and divide the product by 850 times the square of the depth in inches: the quotient will be the breadth in inches.*

Rule 2.—To find the depth of an uniform cast iron beam, to bear a given weight in the middle.

Multiply the length of the bearing in feet, by the weight to be supported in pounds, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the nth part of the depth, n representing any number at will. Then the rule becomes—

Rule 3.—Multiply n times the length in feet, by the weight in pounds; divide this product by 850, and the cube root of the quotient will be the depth required, and the breadth will be the nth part of the depth.

Remark.—The rules are the same for inclined as for

* If the bar be of wrought iron, divide by 952 instead of 850.
If the beam be of oak, divide by 212 instead of 850.
If it be of yellow fir, divide by 255 instead of 850.

horizontal beams, when the horizontal distance between the supports is taken for the length of bearing.

Case 2.

When the weight is not in the middle between the supports.

Rule 4.—Multiply the distance of the weight from one of the supports, by the distance of the weight from the other support; and four times this product, divided by the whole length between the supports, will give the effective leverage of the weight in feet, which being used instead of the length in any of the foregoing rules, the breadth and depth may be found by them.

Case 3.

When the load is uniformly distributed over the length of the beam, the beam being supported at both ends.

Rule 5.—The same rules apply as in Case 1; only, instead of dividing by 850, divide by 1700, which is the double of 850.

Case 4.

Rule 6.—When a beam is fixed at one end, and the load or weight is applied at the other; also, when a beam is supported upon a centre of motion.

In a beam fixed at one end (Fig. 50), take B C for the length; or if the beam be supported in the middle (Fig. 51), take B C or B C′ for the length, and apply the rules given in Case 1; only, instead of dividing by 850, divide by 212.

Note.—For wrought iron taken 238 for a divisor.

If the weight be uniformly distributed over the length of the beam, take 425 as a divisor, instead of 850, in the rules given in Case 1.

Example 1.

What is the breadth of a beam, 30 feet long, 20 inches deep, and which is loaded in the middle with a weight of 20 tons?

$$20 \text{ tons} = 44800 \textit{ lbs}.$$

Then, by Rule 1, $\frac{44800 \times 30}{850 \times 20^2} = 4$ inches nearly.

Example 2.

What is the depth of a beam, 10 feet long and 4 inches broad, so as so sustain a weight of 5 tons at its middle point?

$$5 \text{ tons} = 11200 \textit{ lbs}.$$

By Rule 2, $\frac{11200 \times 10}{850 \times 4} = 33$ nearly, the square root of which is $5\frac{3}{4}$ inches, the depth required.

Example 3.

What are the dimensions of a beam 40 feet long, which is capable of sustaining a weight of 5 tons at its middle point, the depth being 4 times its breadth?

In this example we must take Rule 3.

Here $n = 4$, and 5 tons = 11200 *lbs.*

Then $40 \times 4 = 160 = n$ times the length.

$\frac{11200 \times 160}{850} = 2108\frac{1}{4}$, the cube root of which is 12·8, the depth in inches, and the breadth $= 12·8 \div 4 = 3·2$ inches.

Example 4.

If the depth of a beam be 3 times its breadth, what will these dimensions be, when the whole length of the beam is 20 feet, and a weight of 20 tons is supported at 8 feet from one end?

Here we must take Rules 3 and 4.

$$\frac{12 \times 8 \times 4}{12} = 32 = \text{the effective leverage.}$$

And since the depth is equal to three times the breadth, $n = 3$; and, by Rule 3, $32 \times 3 = 96$.

$\frac{44800 \times 96}{850} = 5059\frac{3}{4}$, the cube root of which is 17, the depth.

Case 5.

When a solid cylinder is supported at the ends, and the weight acts at the middle of the length.

Rule 7.—Multiply the weight in pounds by the length in feet, divide this product by 500, and the cube root of the quotient will be the diameter in inches.*

Case 6.

When the cylinder is supported at the ends, but the strain is not in the middle of the length.

Rule 8.—Multiply the product of the segments into which the strained point divides the beam in feet, by 4 times the weight in pounds; when this product is divided by 500 times the length in feet, the cube root of the quotient will be the diameter in inches.

Case 7.

When the load is uniformly distributed over the length of a solid cylinder supported at the ends.

Rule 9.—Multiply the length in feet by the weight in pounds, and one-tenth of the cube root of the product will be the diameter in inches.

Case 8.

When a cylinder is fixed at one end, and the load applied at the other.

Rule 10.—Multiply the leverage the weight acts with

* For wrought iron, divide by 560 instead of 500.

in feet, by the weight in pounds; the fifth part of the cube root of this product will be the diameter in inches.

Example 1.

Required the diameter of a horizontal shaft of cast iron to sustain a pressure of 1000 *lbs.* in the middle of its length, the length being 10 feet.

$$\frac{1000 \times 10}{500} = 20;$$

and the cube root of 20 is $2\frac{3}{4}$ inches nearly.

Example 2.

What must be the diameter of a cast iron shaft to resist a pressure of 2000 *lbs.* at 2 feet from the end, the whole length of the shaft being 7 feet?

Since the load is applied at 2 feet from one end of the shaft, it must be 5 feet from the other.

$\dfrac{2 \times 5 \times 4 \times 2000}{500 \times 7} = 22{\cdot}8$, the cube root of which is $2\frac{5}{8}$ inches, the diameter required.

ON GUDGEONS.

In gudgeons, one-fifth of the diameter is usually allowed for wear; and, on this principle, Mr. Tredgold gives the following rule:—

Multiply the stress in pounds by the length in inches; and the cube root of the product, divided by 9, is the diameter of the gudgeon in inches.

Example.

If the stress on a gudgeon be 10 tons, and its length 7 inches, what is the diameter?

$$10 \text{ tons} = 22400 \ lbs.$$

$7 \times 22400 = 156800$, the cube root of which is 54 nearly; and $\frac{54}{9} = 6$ inches, the diameter required.

On the Forms of Beams.

86. In the construction of beams, it is necessary that their form should be such that they will be equally strong throughout; or, in other words, they will offer an equal resistance to fracture in all their parts, and will therefore be equally liable to break at one part of their length as at another.

87. If a beam be fixed at one end, and loaded at the other, and the breadth uniform throughout its length, then, that the beam may be equally strong throughout, its form must be that of a parabola.

This form is generally used in the beams of steam-engines; and, in double-acting steam-engines, the beam is strained sometimes from one side, and sometimes from the other; therefore both the sides should be of the same form.

The crank, as used in the steam-engine, should be of the same form.

88. Dr. Young and Mr. Tredgold have considered that it will answer better, in practice, to have some straight-lined figure to include the parabolic form; and the form which they propose is to draw a tangent to the point A of the parabola A C B (Fig. 52). But as few practical men understand how to draw a tangent to a parabola, or even a parabola itself, we will here shew how they may do both.

We will, in the first place shew how to draw a parabola.

Let C B represent the length of the beam, and A B the semi-ordinate, or half the base; then, by the property of the parabola, the squares of all ordinates to the same diameter are to one another as their respective abscissas.

Now, if we take C B = 4 feet, and A B = 1 foot, we may proceed to apply this property to determine the length of the semi-ordinates corresponding to every foot in the length of the beam, as follow:—

$$C B : A B^2 :: C F : E F^2;$$
that is, $48 : 12^2 :: 36 : 108 = E F^2$;
the square root of which is 10·4 nearly $= E F$.
And $C B : A B^2 :: C G : G H^2$;
$$48 : 12^2 :: 24 : 72 = G H^2;$$
the square root of which is 8·5 nearly $= G H$.
$$C B : A B^2 :: C I : I K^2;$$
$$48 : 12^2 :: 12 : 36 = I K^2;$$
the square root of which is 6 inches $= I K$.
Now, if we take $C L = 6$ inches,
then $C B : A B^2 :: C L : L M^2$;
$$48 : 12^2 :: 6 : 18 = L M^2;$$
the square root of which is 4·24, which is very near $4\frac{1}{4}$ inches $= L M$.

Now, if any flexible rod be bent so as just to touch the tops A, E, H, K, M, of the ordinates, and the vertex C, then the form of this rod is a parabola.

To draw a tangent to any point A of a parabola:—

From the vertex C of the parabola draw C D perpendicular to C B, and make it equal to $\frac{1}{2}$ A B; then join A, D, and the right line A D will be a tangent to the parabola at the point A; that is, it touches the parabola at that point.

In the same manner, we may draw a tangent to the parabola at any other point, by erecting a perpendicular at the vertex equal to half the semi-ordinate at that point.

89. When a beam is regularly diminished towards the points that are least strained, so that all the sections are similar figures, whether it be supported at each end and loaded in the middle, or supported in the middle and loaded at each end, the outline should be a cubic parabola.*

90. When a beam is supported at both ends, and is of the same breadth throughout, then, if the load be uniformly distributed throughout the length of the beam, the

* Gregory's Mechanics, Vol. I. Art. 181; or Tredgold on Cast Iron, page 48.

line bounding the compressed side should be a semi-ellipse.

91. The same form should be made use of for the rails of a waggon-way, where they have to resist the pressure of a load rolling over them.*

Beams of Pumping Engines.

By Art. 87, if a beam be fixed at one end and the load applied at the other, or, which is the same thing, if a beam be supported on a centre of motion, then the figure of equal strength is a parabola, the breadth of the beam being the same throughout.

If the beam be fixed in the middle, the following rule may be used:—

Multiply the cube of the length of the beam in feet from the centre of motion, by the weight in pounds; and twice this product, divided by 2662 times the deflection, will give the product of the breadth and cube of the depth in inches.

This rule is for an uniform beam, of which the section is a rectangle.

But if the beam be of a parabolic kind (see Art. 88), divide by 1628 instead of 2662.

Example.

Let it be required to determine the breadth and depth of a beam, in the form of a parabola, for a pumping engine; its whole length being 24 feet, the parts on each side of the centre of motion equal, and the straining force 28000 *lbs.*; the deflection not to exceed $\frac{1}{4}$ of an inch.

$\frac{28000 \times 12^3 \times 2}{1628 \times \frac{1}{4}} = 237759 =$ the product of the breadth and cube of the depth.

* Tredgold, page 49.

If we take the breadth = 3 inches, then $\frac{237759}{3} =$ 79253, the cube root of which is 43 inches, the depth.

If the beam had been uniform, and its section a rectangle,

$$\frac{28000 \times 12^3 \times 2}{2662 \times \frac{1}{4}} = 145406.$$

If the breadth be made 3 inches, then $\frac{145406}{3} = 48468$, the cube root of which is 36·5 nearly, the depth required.

Cranks.

Multiply the weight or power, in pounds, acting at the end of the crank, by the cube of its length in feet; and this product, divided by 2662 times the deflection, will give the product of the breadth and cube of the depth in inches.

Example.

If the force acting upon a crank be 6000 *lbs.* and its length be 3 feet, what are its breadth and depth, so that the deflection may not exceed one-tenth of an inch.

$\frac{6000 \times 3^3}{2662 \times \cdot 1} = 610$ nearly, = breadth multiplied by the cube of the depth.

If the breadth be made 3 inches, the depth should be nearly 6 inches, for the cube of $6 \times 3 = 648$.

If the depth at the end where the force acts be half the depth at the axis, divide by 1628 instead of 2662.

$\frac{6000 \times 3^3}{1628 \times \cdot 1} = 1000$ = breadth multiplied by the cube of the depth.

If we make the breadth 4 inches, then $\frac{1000}{4} = 250$, the cube root of which is 6·3 inches, the depth required.

Wheels.

Multiply the weight or power, in pounds, acting at the end of the arm, by the cube of its length in feet; and this product, divided by 2662 times the number of arms multiplied by the deflection, will give the product of the breadth and cube of the depth.

When the depth at the rim is half that at the axis, divide by 1628 instead of 2662.

Example.

If the force which acts at the circumference of a spur wheel be 1600 *lbs.* the radius of the wheel 6 feet, and the number of arms 8, and let the deflection not exceed $\frac{1}{10}$ of an inch; required the breadth and depth.

$$\frac{1600 \times 6^3}{2662 \times 8 \times \cdot 1} = 162\tfrac{1}{4} = \text{breadth and cube of the depth.}$$

If the breadth be made 2·5 inches, then $\frac{162{\cdot}25}{2{\cdot}5} = 64{\cdot}9$, the cube root of which is 4·018, the depth.

Note.—These rules are formed from the formulæ given in Tredgold's Essay on Cast Iron, 2d edition, pages 204, 206.

On Compression.

The experiments of Mr. Rennie on the resistance to compression in bars of cast iron, of short lengths, are as follow:—

Iron taken from the block, specific gravity 7·033.

On cubes of $\frac{1}{8}$ of an inch.

	lbs. avoirdupois.
Side of cube $\frac{1}{8}$ of an inch was crushed by	1,454
Ditto ... $\frac{1}{8}$ do.	1,416

On cubes of $\frac{1}{4}$ of an inch.

Side of cube $\frac{1}{4}$ of an inch was crushed by	10,561
Ditto ... $\frac{1}{4}$ do.	9,020

Horizontal castings, specific gravity 7·113.

						lbs.
Side of cube	$\frac{1}{4}$	inch was crushed by				10,432
Ditto	...	$\frac{1}{4}$ do.	...	...	...	10,720
Ditto	...	$\frac{1}{4}$ do.	...	...	...	10,605
Ditto	...	$\frac{1}{4}$ do.	...	...	...	8,699

The mean of which is 10,114 *lbs.*

Vertical castings, specific gravity 7·074.

						lbs.
Side of cube	$\frac{1}{4}$	inch was crushed by				12,665
Ditto	...	$\frac{1}{4}$ do.	...	...	...	10,950
Ditto	...	$\frac{1}{4}$ do.	...	...	...	11,088
Ditto	...	$\frac{1}{4}$ do.	...	...	...	9,844

The mean of which is 11136·75 *lbs.*

On pieces of different lengths.

						lbs.
Area	$\frac{1}{8} \times \frac{1}{8}$,	length $\frac{3}{4}$ inch,	was crushed by			1743
Do.	$\frac{1}{8} \times \frac{1}{8}$,	do. 1 do.		...	...	1439
Do.	$\frac{1}{4} \times \frac{1}{2}$,	do. $\frac{1}{2}$ do.		...	...	9374
Do.	$\frac{1}{4} \times \frac{1}{4}$,	do. 1 do.		...	...	6321

A prism having a logarithmic curve for its limits, resembling a column, and being a quarter of an inch in diameter by one inch long, broke with 6954 *lbs.*

Some of the most useful cases of this kind of strain are the piston-rods of steam-engines.

Mr. Tredgold shews, that in a double acting steam-engine, the pressure on each square inch of the piston being 10 pounds, the diameter of the piston-rod should be one-fifteenth of the diameter of the cylinder; and, in practice, it is usual to make it one-tenth, to allow for wear.

In single acting engines, the pressure on the square inch being 11 pounds, to allow for wear, the piston-rod is made one-fifteenth of the diameter of the cylinder.

On Torsion.

93. The resistance which a shaft or any other body offers to a force applied to twist it round, is called the resistance to torsion.

94. The strength of cylinders to resist torsion or twisting, is generally estimated to be proportional to the cubes of the diameters.

In the Additions to Buchanan on Mill-work, by Mr. Tredgold, the following formula is given for cylindrical shafts of cast iron to resist torsion:—

$$\sqrt[3]{\frac{240\,h}{n}} = d;$$

where h denotes the number of horses' power, n the number of revolutions of the shaft per minute, and d its diameter in inches.

This expressed in words—Multiply the number of horses' power by 240, and divide this product by the number of revolutions of the shaft per minute; then, the cube root of the quotient will give the diameter of the shaft in inches.

Example.

If a cast iron shaft make 30 revolutions per minute, and the moving power equal to 20 horses, required the diameter of the shaft to resist the torsion.

By the rule, $\frac{20 \times 240}{30} = 160$, the cube root of which is $5\frac{1}{2}$ inches nearly, the diameter required.

If the shaft have both to sustain lateral pressure and torsion, the formula is,

$$\sqrt[3]{\left(\frac{240\,\mathrm{H}}{\mathrm{N}} + \frac{\mathrm{W}\,l^2}{2}\right)} = d, \text{ the diameter in inches.}$$

And l is the length in feet between the bearings, H the number of horses' power which are equal to the first mover,

N the number of revolutions to be made by the shaft per minute, and W the lateral stress in *cwts.* (See Tredgold's Additions to Buchanan on Mill-work, page 382.)

Example.

If a cylindrical shaft of cast iron make 40 revolutions per minute, the moving power being equal to 7 horses, the length of the shaft 10 feet, and the lateral stress 6 *cwt.* required the diameter of the shaft.

By the formula, $\sqrt[5]{\left(\frac{240 \times 7}{40} + \frac{6 \times 10^2}{2}\right)} = \sqrt[5]{(42 + 300)} = \sqrt[5]{342} = 7$ inches nearly.

95. It is necessary, before taking leave of this subject, to call the attention of the practical mechanic to the distinction between stiffness and strength.

96. Stiffness may be defined to be that property which resists flexure or bending; and strength has already been shewn to be that which resists fracture. Therefore, the limit of stiffness is flexure, and the limit of strength is fracture.

97. Beams of equal lengths have their lateral stiffness to bear a load at any point in the length, as the breadth and cube of the depth.* But their lateral or transverse strengths are as the breadth and square of the depth, by Art. 84.

Thus, if a beam be two feet square, it will sustain sixteen times as much weight, without bending, as a beam one foot square.

But a beam two feet square will only be eight times stronger than a beam one foot square, the beams being of the same length.

98. Beams of different lengths have their stiffness (to bear a load at any point of their lengths) directly as the breadth and the cube of the depth, and inversely as the cube of

* Dr. Young's Natural Philosophy, vol. i. p. 139.

the length; and have their strength directly as the breadth and the square of the depth, and inversely as the length.*

It may be necessary to remark here, that the rules for the strength, &c. of timber, are taken from Barlow's Essay, 2d edition; and those on cast iron from Tredgold's Essay, 2d edition.

We will afterwards give tables on the strength of cast iron shafts.

DYNAMICS.

99. Dynamics treats of the various effects of bodies in motion, and is the most elementary branch of that doctrine, and the most general in its principles.

We may here entirely abstract from the consideration of the figure of the moving bodies, and treat of them as if their matter were collected in mere physical points.

On Matter and Motion.

100. The quantities of matter in all bodies are in the compound ratio of their magnitudes and densities.

For if the magnitudes are equal, the quantities of matter will be as the densities; and if the densities are equal, the quantities of matter will be as the magnitudes. Therefore, the quantities of matter are universally in the compound ratio of both.

Cor. 1. The quantities of matter in all similar bodies are as the densities and cubes of the diameters.

* Dr. Young's Natural Philosophy, vol. ii. Art. 333 and 335.

For the magnitudes are as the cubes of the diameters, or other like dimensions.

Cor. 2. The masses, or quantities of matter, are as the magnitudes and specific gravities.

For, by Art. 4 and 13, the densities of bodies are as the specific gravities.

Therefore, if B denote the quantity of matter in a body, M its magnitude, D its density, S its specific gravity, and P its diameter; and let b denote the quantity of matter in any other body, m its magnitude, d its density, s its specific gravity, and p its diameter, or other like dimension; then,

$$B : b :: M \times D : m \times d :: M \times S : m \times s :: P^3 \times D : p^3 \times d$$

$$M : m :: \frac{B}{D} : \frac{b}{d} :: \frac{B}{S} : \frac{b}{s} :: P^3 : p^3$$

$$D : d :: \frac{B}{M} : \frac{b}{m} :: \frac{M \times S}{P^3} : \frac{m \times s}{p^3} :: S : s$$

$$S : s :: \frac{B}{M} : \frac{b}{m} :: \frac{D \times P^3}{M} : \frac{d \times p^3}{m} :: D : d$$

$$P^3 : p^3 :: \frac{B}{D} : \frac{b}{d} :: \frac{M \times S}{D} : \frac{m \times s}{d} :: M : m$$

Uniform Motion.

101. The velocity of a moving body is said to be uniform, when the body passes over equal spaces in equal times.

102. The space which a body moving with an uniform velocity passes over in any time, is found by multiplying the time by the velocity; that is, by multiplying the number of seconds the body has been in motion, by the space moved over in one second.

Thus, if a body moves uniformly at the rate of 4 feet per second, and is 30 seconds in motion, then $4 \times 30 = 120$ feet, the space or length of line passed over by the body.

Now, if S be the length of the line described, in the time T, with the uniform velocity V, then,

$$S = V \times T$$
$$V = \frac{S}{T}$$
$$T = \frac{S}{V}$$

If two bodies move uniformly for 4 and 6 seconds, the former with a velocity of 10 feet, and the latter with a velocity of 5 feet per second; then the spaces described by them will be in proportion to each other as 10×4 to 5×6, or as $4 : 3$.

Momentum.

103. When the motion of a body is considered with respect to the mass, or quantity of matter, moved, as well as its velocity, it is called its momentum, or quantity of motion.

104. The momentum of a body is in the compound ratio of its quantity of matter and velocity.

For the momentum of the whole body is the sum of the motions of all its parts; therefore the quantity of motion depends on the number of parts and the velocity of each.

Let M = the momentum of a body, W its quantity of matter or weight, and V its velocity; and let m = the momentum, w = the weight, and v = the velocity of any other body, expressed in the same terms; then the relation which exists between M, W, V, and m, w, v, may be expressed as follows:—

$$M : m :: W \times V : w \times v$$
$$W : w :: \frac{M}{V} : \frac{m}{v}$$
$$V : v :: \frac{M}{W} : \frac{m}{w}$$

If the bodies are equal, we have,

$$M : m :: V : v$$

If the bodies move with equal velocities, then,

$$M : m :: W : w$$

If two bodies move with velocities which are inversely as their quantities of matter, their momenta will be equal.

$$\text{For then, } M : m :: v : V$$

And since the product of the extreme terms of a proportion is equal to the product of the mean terms, we have

$$M V = m v$$

If a body, the weight of which is 8 *lbs.* moves with a velocity of 10 feet per second; and another body, the weight of which is 10 *lbs.* moves with a velocity of 5 feet per second; then,

$$M : m :: 8 \times 10 : 10 \times 5 :: 8 : 5$$

That is, the momentum of the former is to the momentum of the latter as 8 to 5.

Or, if we take the momentum of the former to that of the latter as 8 to 5, and their velocities as 2 to 1, then their weights are to each other as $\frac{8}{2}$ to $\frac{5}{1}$, that is as 4 to 5.

And if the momenta of four bodies are as 1, 2, 3, 4, and their weights as 3, 4, 5, 6, then their velocities are as $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}$; or as $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}$; or, which is the same, 10, 15, 18, 20.

The battering-ram of Vespasian weighed, suppose, 100,000 *lbs.*; and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time; and this was sufficient to demolish the walls of Jerusalem: with what velocity must a bullet, which weighs but 30 *lbs.* be moved, in order to do the same execution?

Here, $100{,}000 \times 20 = 2{,}000{,}000$ *lbs.* the momentum of the ram; and, to produce the same effect, the momentum of the bullet must be equal to the momentum of the ram. Therefore, $2{,}000{,}000 \div 30 = 66{,}666\frac{2}{3}$ feet per second.

ACCELERATED MOTION.

105. A force acting incessantly upon a body is called a constant or uniformly accelerating force, when the velocity increases equally in equal times. Thus, the force of gravity near the earth's surface is of this kind; for it generates a velocity of $32\frac{1}{6}$ feet in each second of time. That is, a body, after falling one second, acquires a velocity of $32\frac{1}{6}$ feet; after falling 2 seconds, it will acquire a velocity of $2 \times 32\frac{1}{6}$ feet; after 3 seconds, a velocity of $3 \times 32\frac{1}{6}$ feet; and so on.

106. It is called a variable force when the variation of the acquired velocity is not the same in each succeeding instant.

Universal Gravitation furnishes us with an example of this kind; for the variation of its law is inversely as the square of the distance from the earth's centre.

Thus, a body at the distance of two semi-diameters from the earth's centre, will be acted on by a force which is only one-fourth part of the force which acts upon a body at the earth's surface, and which will therefore generate only one-fourth part of the velocity. At three semi-diameters, the force will be one-ninth; at four semi-diameters, one-sixteenth; and so on, always decreasing inversely as the square of the distance.

107. The momentum, or quantity of motion, which is generated by an uniformly accelerating force, is in the compound ratio of the force and time of acting.

For, in any given time, the momentum which is generated will be proportional to the force which generates it. And since the force has the same efficacy in each instant of time, the whole momentum will be as the sum of these instants, or whole time. Consequently, the whole momentum, or quantity of motion generated, is in the compound ratio of the force and time of acting.

Cor. 1. The quantity of motion lost or destroyed in any time, by a force acting in an opposite direction, is also in the compound ratio of the force and time.

Cor. 2. The velocity generated or destroyed in any time, is as the force and time directly, and quantity of matter reciprocally; that is, v is as $\frac{ft}{b}$; where v denotes the velocity, f the force, and b the body, or quantity of matter. For, by Art. 104, the momentum is as the quantity of matter and velocity; therefore, the velocity is as the momentum directly and quantity of matter reciprocally; that is, by this Art. as the force and time directly and quantity of matter reciprocally.

Cor. 3. Hence, if the body be given, the velocity will be in the compound ratio of the force and time; and if the force be given, the time is in the compound ratio of the quantity of matter and the velocity, or as the momentum.

108. If a given body be urged by a constant and uniform force, the space which is described by the body from the beginning of the motion is as the force and square of the time.

For, suppose the time to be divided into an indefinite number of equal parts. Then, in each of these equal parts of time, the space described will be as the velocity gained; that is, by Art. 107, Cor. 3, as the force and time from the beginning. And the sum of all the spaces, or the whole space described, will be as the force and the sum of all the equal parts of time from the beginning. If we put $n =$ the whole time, the whole space described will be as the sum of the times $1 + 2 + 3 + 4$, &c. to n. But the sum of the arithmetical series, $1 + 2 + 3 + 4$, &c. to $n = \frac{1+n}{2} \times n$; but n is infinite, therefore, $\frac{1+n}{2} \times n$ becomes $\frac{n^2}{2}$. Hence, the whole space described will be as the force and $\frac{1}{2} n^2$, or force and square of the time; for $\frac{1}{2}$ is a constant quantity.

Cor. 1. If a body, which is urged by a constant and uniform force, move through any space, it will move through twice that space in the same time by the velocity acquired. For the sum of all the spaces described by that force, $1 + 2 + 3$, &c. to n, was shewn to be $\frac{1}{2} n^2$; and the sum of all the spaces described by the last velocity will be $n + n + n$, &c. to n terms, the sum of which is n^2. But n^2 is double of $\frac{1}{2} n^2$; therefore the space described by the last velocity is double the space described by the accelerating force.

Cor. 2. If a body is acted on by a constant and uniform force, the space described from the beginning of the motion is as the velocity acquired and the time of moving.

Cor. 3. Universally, in all bodies urged by any constant and uniform forces, during any times, the spaces passed over are as the forces and squares of the times directly, and the quantity of matter reciprocally.

Cor. 4. The product of the force and square of the time, is as the product of the body and space described.

Cor. 5. The product of the force and time is as the product of the quantity of matter and velocity.

Cor. 6. The product of the body and the square of its velocity, is as the product of the force and space described.

Scholium.

109. If a body be acted on by any two constant and uniformly accelerating forces, f, F, during the respective times t and T, at the end of which are generated the velocity is v, V, and describes the spaces s and S, then,

$$s : S :: t v : T V$$

$$\therefore \frac{s}{S} = \frac{t v}{T V}$$

$$s : S :: f t^2 : F T^2$$

$$\therefore \frac{s}{S} = \frac{f t^2}{F T^2}$$

$$s : S :: \frac{v^2}{f} : \frac{V^2}{F}$$

$$v : V :: f t : F T$$

$$\therefore \frac{v}{V} = \frac{f t}{F T}$$

$$v : V :: \frac{s}{t} : \frac{S}{T}$$

$$\therefore \frac{v}{V} = \frac{s T}{S t}$$

$$v : V :: \sqrt{f s} : \sqrt{F S}$$

$$\therefore \frac{s}{S} = \frac{v^2 F}{V^2 f}$$

$$t : T :: \frac{v}{f} : \frac{V}{F}$$

$$\therefore \frac{t}{T} = \frac{F v}{f V}$$

$$t : T :: \frac{s}{v} : \frac{S}{V}$$

$$\therefore \frac{t}{T} = \frac{s V}{S v}$$

$$t : T :: \sqrt{\frac{s}{f}} : \sqrt{\frac{S}{F}}$$

$$\therefore \frac{t}{T} = \sqrt{\frac{F s}{f S}}$$

$$\therefore \frac{v}{V} = \sqrt{\frac{f s}{F S}}$$

$$f : F :: \frac{v}{t} : \frac{V}{T}$$

$$\therefore \frac{f}{F} = \frac{v T}{V t}$$

$$f : F :: \frac{s}{t^2} : \frac{S}{T^2}$$

$$\therefore \frac{f}{F} = \frac{s T^2}{S t^2}$$

$$f : F :: \frac{v^2}{s} : \frac{V^2}{S}$$

$$\therefore \frac{f}{F} = \frac{v^2 S}{V^2 s}$$

Collecting these:—

$$\frac{s}{S} = \frac{t v}{T V} = \frac{f t^2}{F T^2} = \frac{v^2 F}{V^2 f}$$

$$\frac{v}{V} = \frac{f t}{F T} = \frac{s T}{S t} = \sqrt{\frac{f s}{F S}}$$

$$\frac{t}{T} = \frac{F v}{f V} = \frac{s V}{S v} = \sqrt{\frac{F s}{f S}}$$

$$\frac{f}{F} = \frac{v T}{V t} = \frac{s T^2}{S t^2} = \frac{v^2 S}{V^2 s}$$

Now, if one of the forces be the force of gravity at the earth's surface, and be represented by unity or 1, and the time $T = 1''$; then the corresponding space S has been found by experiment to be $16\frac{1}{12}$ feet, and therefore its velocity $V = 2\,S = 32\frac{1}{6}$ feet.

Above, substitute 1 in the place of F, $1''$ in the place of T, $16\frac{1}{12}$ feet in the place of S, and $32\frac{1}{6}$ feet in the place of V; and we have,

$$s = \tfrac{1}{2}\, t\, v = 16\tfrac{1}{12} f\, t^2 = \frac{v^2}{64\frac{1}{3} f}$$

$$v = \frac{2\, s}{t} = 32\tfrac{1}{6} f\, t = \sqrt{64\tfrac{1}{3} f\, s}$$

$$t = \frac{2\ s}{v} = \frac{v}{32\frac{1}{6}\ f} = \sqrt{\frac{s}{16\frac{1}{12}\ f}}$$

$$f = \frac{v}{32\frac{1}{6}\ t} = \frac{s}{16\frac{1}{12}\ t^2} = \frac{v^2}{64\frac{1}{3}\ s}$$

These theorems will be expressed in words afterwards, when we come to treat of the motion of bodies on inclined planes.

When any quantity or quantities are given, or are always the same, they must be left out; thus,

By Art. 108, Cor. 3, $s : S :: \frac{f\,t^2}{b} : \frac{F\ T^2}{B}$; but in this Art. the body is constant, therefore $b = B$, and in this case,

$s : S :: \frac{f\,t^2}{b} : \frac{F\ T^2}{b}$; or, which is the same, $s : S :: f\,t^2 : F\ T^2$.

Also, such quantities as are proportional to each other must be left out. Thus, if the quantity of matter be proportional to the force, as all bodies are in respect to their gravity,* then the space described is as the square of the velocity. The space is also as the square of the time. Hence the velocity is as the time.

We may now show the application of these proportions to falling bodies.

* The weights of all bodies in the same place are proportional to the quantities of matter they contain, without any regard to their bulk, figure, or kind. For twice the matter will be twice as heavy, thrice the matter thrice as heavy, and so on.

ON GRAVITY.

110. Gravity is that power or force which causes bodies to approach each other. This universal principle, which pervades the whole system of nature, may be enunciated as follows.

The mutual tendency of two bodies towards each other increases in the same proportion as their masses are increased, and the square of their distance is decreased; and it decreases in proportion as their masses are decreased, and as the square of their distance is increased.

Thus, if any two bodies A and B be placed in free space at a given distance from each other, and if we suppose the mass of A to be double the mass of B, then B will move towards A with double the velocity with which A moves towards B.

Also, if at any given distance A tends towards B with a given force, at double that distance A will tend towards B with one-fourth of that force, at treble the distance with one-ninth of that force, and so on.

Philosophers have formed theories of various kinds to account for this universal principle. Some have considered that it is produced by particles emanating from a centre or centres. But, as Dr. Paley* very justly observes, we are totally at a loss to comprehend how particles streaming from a centre can possibly draw bodies toward that centre. The impulse, if impulse there be, is all the other way.

* We would particularly recommend to our readers the perusal of Paley's Natural Theology, it being a work replete with useful information.

We are equally at a loss if we consider that the effect is produced by a conflux of particles flowing towards a centre, and carrying down all bodies along with it; for, if such a fluid exists, it must act very powerfully, and at the same time offer no resistance whatever to bodies moving in it, which is contrary to the known constitution of fluids.

We are also utterly unable to conceive how one body can act upon another at a distance, or, in other words, that a body can act where it is not; and it appears no more ridiculous to assert that a body can act when it ceases to exist, than to assert that a body can act where it does not exist.

Indeed, every hypothesis relating to the cause of gravity is embarrassed with insuperable difficulties. It seems, as it were, to be among the arcana of the Almighty; and, until a theory can be advanced which can clear up all these difficulties, it is more prudent to conclude, with the illustrious Newton, "that in absence of the secondary cause of gravity, we may attribute it to the final cause of all things, the finger of God, the constant impression of divine power."

Terrestrial Gravity.

111. Terrestrial Gravity is that force by which bodies are urged towards the centre of the earth, and it is measured by the velocity generated in a second of time. As has been already remarked, experiments shew that a falling body describes $16\frac{1}{12}$ feet in the first second, and it has then acquired a velocity of $32\frac{1}{6}$ feet, which is therefore the true measure of the force of gravity.

There appears to be an inequality of the action of gravity upon different kinds of matter near the surface of the earth. But this arises entirely from the resistance which they meet with in passing through the air; for, in the exhausted receiver of an air-pump, all bodies fall equally; a guinea acquires no greater velocity than a feather; an

ounce of feathers and a ton of gold fall through equal spaces in equal times, and will reach the bottom of the receiver exactly at the same instant of time.

112. Since it is known by experiment that the space which a body describes by falling freely from rest is $16\frac{1}{12}$ feet in the first second of its fall, by Art. 108, Cor. 1, the body will have acquired a velocity of $32\frac{1}{6}$ feet; that is, the body will have acquired a velocity which, if continued uniformly, would carry it through twice the space in the same time, if gravity ceased at that instant to act.

Put s for the space described by the body in any other time t, and v the velocity acquired; and since, by the latter part of Art. 109, the spaces are as the squares of the velocities,

$$s : 16\tfrac{1}{12} :: v^2 : (32\tfrac{1}{6})^2 = 4\,(16\tfrac{1}{12})^2 *$$

$$\therefore s = \frac{16\frac{1}{12}\, v^2}{4\,(16\frac{1}{12})^2} = \frac{v^2}{64\frac{1}{3}};\ \text{and } v = 2\sqrt{16\tfrac{1}{12}\, s}$$

The spaces are also as the squares of the times.

$$s : 16\tfrac{1}{12} :: t : 1^2$$

$$\therefore s = 16\tfrac{1}{12}\, t^2;\ \text{and } t = \sqrt{\frac{s}{16\frac{1}{12}}}$$

Also, the velocities are as the times; that is, the velocity varies as the time varies.

$$t : 1 :: v : 32\tfrac{1}{6}$$

$$\therefore v = 32\tfrac{1}{6}\, t;\ \text{and } t = \frac{v}{32\frac{1}{6}}$$

Collecting these:

$$s = \frac{v^2}{64\frac{1}{3}} = 16\tfrac{1}{12}\, t^2 = \frac{t\,v}{2}$$

$$v = 2\sqrt{16\tfrac{1}{12}\, s} = 32\tfrac{1}{6}\, t = \frac{2\,s}{t}$$

$$t = \sqrt{\frac{s}{16\frac{1}{12}}} = \frac{v}{32\frac{1}{6}} = \frac{2\,s}{v}$$

* The square of any quantity is equal to four times the square of half that quantity; thus $16\frac{1}{12}$ is half of $32\frac{1}{6}$; hence $4\,(16\frac{1}{12})^2 = (32\frac{1}{6})^2$.

113. Also, since the spaces described by falling bodies are as the squares of the times, if those times be represented by the numbers 1, 2, 3, 4, &c. the spaces described in those times will be as 1, 4, 9, 16, &c. which are the squares of 1, 2, 3, 4, &c. respectively; and the spaces described in a series of equal portions of time will be as the odd numbers 1, 3, 5, 7, &c.; that is, if a body fall through $16\frac{1}{12}$ feet in the first second, it will fall through $3 \times 16\frac{1}{12}$ in the next second, $5 \times 16\frac{1}{12}$ in the third, and so on.

Also, the velocities are as the numbers 1, 2, 3, 4, &c. since the velocities are as the times.

The above may be expressed in the form of rules, for the use of those who do not understand the application of formulæ, in the following manner:

Given the velocity which a heavy body acquires in falling, to find the space it has fallen through to acquire that velocity.

Rule 1.—Divide the square of the acquired velocity by $64\frac{1}{3}$, and the quotient will give the space in feet which the body has fallen through to acquire that velocity.

Example.

How far must a body fall to acquire a velocity of 120 feet per second?

$$120^2 = 14400, \text{ and } 14400 \div 64\tfrac{1}{3} = 223{\cdot}8 \text{ feet.}$$

By the formula,

$$s = \frac{v^2}{64\frac{1}{3}} = \frac{120^2}{64\frac{1}{3}} = \frac{14400}{64\frac{1}{3}} = 223{\cdot}8 \text{ feet.}$$

Given the time a heavy body has been in falling to find the space fallen through.

Rule 2.—Multiply the square of the time in seconds by $16\frac{1}{12}$, and the product will give the space fallen through in feet.

Example.

Through what space will a body fall in 10 seconds?

$10^2 = 100$, and $100 \times 16\frac{1}{12} = 1608\frac{1}{3}$ feet.

By the formula,

$s = 16\frac{1}{12} \times t^2 = 16\frac{1}{12} \times 10^2 = 1608\frac{1}{3}$, the same as before.

Given the space which a heavy body has fallen through to find the velocity acquired.

Rule 3.—Multiply the space fallen through in feet by $16\frac{1}{12}$, and twice the square root of the product will give the velocity acquired in feet.

Example.

What velocity will a body acquire in falling 100 feet?

$100 \times 16\frac{1}{12} = 1608{\cdot}3$, the square root of which is $40{\cdot}104$, and $40{\cdot}104 \times 2 = 80{\cdot}208$ feet per second.

By formula,

$v = 2\sqrt{16\frac{1}{12}\ s} = 2\sqrt{16\frac{1}{12} \times 100} = 2\sqrt{1608{\cdot}3} =$ $80{\cdot}208$ feet per second.

Given the time which a heavy body has been in falling to find the velocity it has acquired.

Rule 4.—Multiply the time of falling in seconds by $32\frac{1}{6}$, and this product will give the velocity acquired in feet.

Example.

What velocity will a body acquire by falling 10 seconds?

$10 \times 32\frac{1}{6} = 321\frac{2}{3}$ feet, the velocity per second.

By formula,

$v = 32\frac{1}{6}\ t = 10 \times 32\frac{1}{6} = 321\frac{2}{3}$, as before.

To find the time which a heavy body will be in falling through a given space.

Rule 5.—Divide the space in feet by $16\frac{1}{12}$, and the square root of the quotient will give the required time in seconds.

Example.

How long will a body be in falling through the space of 100 feet?

$100 \div 16\frac{1}{12} = 6\cdot217$, the square root of which is $2\cdot5$ seconds nearly.

By formula,

$$t = \sqrt{\frac{s}{16\frac{1}{12}}} = \sqrt{\frac{100}{16\frac{1}{12}}} = 2\cdot5 \text{ seconds nearly.}$$

To find the time which a heavy body must fall to acquire a given velocity.

Rule 6.—Divide the given velocity in seconds by $32\frac{1}{6}$, and this quotient is the time in seconds which a body must fall to acquire that velocity.

Example.

How long must a body fall to acquire a velocity of 140 feet per second?

$$140 \div 32\tfrac{1}{6} = 4\cdot35 \text{ seconds.}$$

By formula,

$$t = \frac{v}{32\frac{1}{6}} = \frac{140}{32\frac{1}{6}} = 4\cdot35 \text{ seconds, as before.}$$

114. In the descent of bodies, gravity generates equal velocities in equal times; and in the ascent of bodies, gravity destroys equal velocities in equal times: therefore if a body be projected upwards with a given velocity, it will rise to the same height from which it must fall to acquire that velocity.

115. If a body be projected either upwards or downwards with a given velocity, to find the space described,

First, suppose the body to be projected downwards with a velocity v, and let t be the time the body is in motion; then the space passed over by the velocity v, continued uniformly for the time t, is $t \times v$; and the space the body will fall through in the time t, by the action of gra-

vity, is $16\frac{1}{12}\,t^2$; therefore the whole space described in the time t is $t \times v + 16\frac{1}{12} \times t^2$.

But if the body is projected upwards, then the distance of the body from the point of projection is $t \times v - 16\frac{1}{12} \times t^2$; for gravity, in this case, acts in an opposite direction to the motion of the body. Hence we have the following rules:—

Case 1.

When a body is projected downwards with a given velocity, multiply the square of the time in seconds by $16\frac{1}{12}$, and the velocity of projection in feet by the number of seconds the body is in motion; then the sum of these products will give the space moved through by the body.

Case 2.

If the body is projected upwards, then the difference of the above products will give the distance of the body from the point of projection.

Example 1.

If a body be projected downwards with a velocity of 20 feet per second, through what space will it fall in 6 seconds?

Now, $6^2 = 36$, and $16\frac{1}{12} \times 36 = 579$ feet, the space passed through by the action of gravity.

Then, $20 \times 6 = 120$ feet, the space passed through by the uniform impulse.

Hence the whole space is $579 + 120 = 699$ feet.

By the formula:

Here, $v = 20$ and $t = 6$; $\therefore t \times v + 16\frac{1}{12} \times t^2 = 6 \times 20 + 16\frac{1}{12} \times 6^2 = 699$ feet.

Example 2.

If a body is projected upwards with the velocity of 250 feet per second, how far will it rise in 2 seconds?

$250 \times 2 = 500$ feet, the space which the body would pass over if gravity did not act.

Then, $2^2 = 4$, and $16\frac{1}{12} \times 4 = 64\frac{1}{3}$, the retardation arising from gravity.

Hence, $500 - 64\frac{1}{3} = 435\frac{2}{3}$ feet.

By the formula:

Here, $v = 250$, and $t = 2$; $\therefore\ t \times v - 16\frac{1}{12} \times t^2 = 2 \times 250 - 16\frac{1}{12} \times 2^2 = 435\frac{2}{3}$ feet.

Example 3.

If a body be projected upwards with a velocity of 30 feet per second, through what space will it ascend before it begins to return?

It is evident, by Art. 114, that if a body be projected upwards with a given velocity, it will ascend to the same height from which it must fall to acquire that velocity; therefore Rule 1 applies to this problem; that is, divide the square of the velocity of projection in feet by $64\frac{1}{3}$, and the quotient will give the height to which the body will ascend.

$30^2 = 900$ feet, and $900 \div 64\frac{1}{3} = 14$ feet nearly.

Example 4.

If a body be projected vertically upwards with a velocity of 100 feet per second, it is required to find the place of the body at the end of 10 seconds.

By the Rule, $100 \times 10 = 1000$ feet, the space which a body would move through if gravity did not act; and $16\frac{1}{12} \times 10^2 = 1608\frac{1}{3}$, the retardation arising from gravity.

Hence, $1000 - 1608\frac{1}{3} = -608\frac{1}{3}$ feet, the negative sign shows that the body will be $608\frac{1}{3}$ feet below the point of projection.

On the Motion of Bodies on Inclined Planes.

116. In treating of the equilibrium of an inclined plane, it was shown that the force on an inclined plane bears the same proportion to the force of gravity, as the height of the plane bears to its length; that is, the force which accelerates the motion of a body down an inclined plane, is that

fractional part of the force of gravity which is represented by the height of the plane divided by its length. Therefore, if h represents the height of the plane, and l its length, then $\frac{h}{l}$ will represent the accelerating force. In the formulæ, pages 82 and 83, for f put $\frac{h}{l}$; or, which is the same, substitute $sin.\ i$ for f, i being the angle of inclination; and we have,

$$s = \tfrac{1}{2}\, t\, v = \frac{16\frac{1}{12}\, h\, t^2}{l} = \frac{l\, v^2}{64\frac{1}{3}\, h}$$

$$v = \frac{2\,s}{t} = \frac{32\frac{1}{6}\, h\, t}{l} = \sqrt{\frac{64\frac{1}{3} \,.\, h\, s}{l}}$$

$$t = \frac{2\,s}{v} = \frac{l\, v}{32\frac{1}{6}\, h} = \sqrt{\frac{l\, s}{16\frac{1}{12}\, h}}$$

$$\frac{h}{l} \text{ or } sin.\ i = \frac{v}{32\frac{1}{6}\, t} = \frac{s}{16\frac{1}{12}\, t^2} = \frac{v^2}{64\frac{1}{3} \,.\, s}$$

$$s = sin.\ i \times 16\tfrac{1}{12}\, t^2 = \frac{v^2}{64\frac{1}{3}\ sin.\ i}$$

$$v = sin.\ i \times 32\tfrac{1}{6}\, t = \sqrt{sin.\ i \times 64\tfrac{1}{3}\, s}$$

$$t = \frac{v}{32\frac{1}{6} \,.\, sin.\ i} = \sqrt{\frac{s}{16\frac{1}{12} \,.\, sin.\ i}}$$

Given the length and height of an inclined plane, to find the space which a body will move through in a given time.

Rule 1.—Multiply the height of the plane in feet by the square of the given time in seconds, and divide this product by the length of the plane, also in feet; and this quotient, multiplied by $16\frac{1}{12}$, will give the space in feet descended or ascended.

Example.

The length of an inclined plane is 100 feet, and the height 50 feet; what space will a body descend through in 3 seconds?

$50 \times 3^2 = 450$, then $450 \div 100 = 4\frac{1}{2}$, and $16\frac{1}{12} \times 4\frac{1}{2} = 72{\cdot}07$ feet, the space required.

By formula,

$$s = \frac{16\frac{1}{12} \times h\,t^2}{l} = \frac{16\frac{1}{12} \times 50 \times 3^2}{100} = 72{\cdot}07 \text{ feet.}$$

Given the length and height of an inclined plane, to find the space which a body must move through to acquire a given velocity.

Rule 2.—Multiply the length of the plane in feet by the square of the acquired velocity; and this product, divided by $64\frac{1}{3}$ times the height of the plane, also in feet, will give the space moved through to acquire that velocity.

Example.

What space must a body descend along the above plane to acquire a velocity of 10 feet per second?

$100 \times 10^2 = 10000$, then $10000 \div 64\frac{1}{3} \times 50 = 3{\cdot}1$ feet, the space required.

By formula,

$$s = \frac{l\,v^2}{64\frac{1}{3}\,h} = \frac{100 \times 10^2}{64\frac{1}{3} \times 50} = \frac{10000}{3216\frac{2}{3}} = 3{\cdot}1 \text{ feet, the}$$

space required.

Given the length and height of an inclined plane, to find the velocity acquired in a given time.

Rule 3.—Multiply $32\frac{1}{6}$ times the height of the plane in feet by the time in seconds; and this product, divided by the length of the plane in feet, will give the velocity acquired.

Example.

The length and height of the inclined plane remaining the same, what velocity will a body acquire in 2 seconds?

$32\frac{1}{6} \times 50 \times 2 = 3216\frac{2}{3}$, and $3216\frac{2}{3} \div 100 = 32\frac{1}{6}$ feet per second.

By formula,

$$v = \frac{32\frac{1}{6}\,h\,t}{l} = \frac{32\frac{1}{6} \times 50 \times 2}{100} = 32\frac{1}{6} \text{ feet, the same as}$$

before.

But this may be seen from other considerations, for $\frac{h}{l} = \frac{50}{100} = \frac{1}{2}$; that is, the accelerating force on the plane is only half the accelerating force of gravity; it will therefore only generate half the velocity, or it will generate in two seconds the same velocity which the force of gravity generates in one second.

Given the length and height of an inclined plane, to find the velocity which a body will acquire in descending through a given space.

Rule 4.—Multiply $64\frac{1}{3}$ times the height of the plane by the space described, all in feet; and if this product be divided by the length of the plane, also in feet, the square root of the quotient will give the velocity acquired in feet.

Example.

What velocity will a body acquire in descending down an inclined plane, the length of which is 20 feet and height 1 foot?

$64\frac{1}{3} \times 1 = 64\frac{1}{3}$, then $64\frac{1}{3} \times 20 = 643\frac{1}{3}$, and $643\frac{1}{3} \div 20 = 64\frac{1}{3}$, the square root of which is 8·02 nearly, the acquired velocity.

By formula,

$$v = \sqrt{\frac{64\frac{1}{3} \times h\,s}{l}} = \sqrt{\frac{64\frac{1}{3} \times 1 \times 20}{20}} = \sqrt{64\tfrac{1}{3}} = 8{\cdot}02$$

feet, as before.

Given the length and height of an inclined plane, to find the time in which a body will acquire a given velocity.

Rule 5.—Multiply the length of the plane in feet by the given velocity in feet; and this product, divided by $32\frac{1}{6}$ times the height in feet, will give the time in seconds.

Example.

If the height of an inclined plane be 1 foot, and the length 40 feet, in what time will a body descending down

this plane by the force of gravity acquire a velocity of 5 feet per second?

$40 \times 5 = 200$, and $200 \div 32\frac{1}{6} \times 1 = 6{\cdot}22$ seconds nearly.

By the formula,

$$t = \frac{l\,v}{32\frac{1}{6}\,h} = \frac{40 \times 5}{32\frac{1}{6} \times 1} = 6{\cdot}22 \text{ seconds.}$$

Given the length and height of an inclined plane, to find the time in which a body will describe a given space.

Rule 6.—Multiply the length of the plane in feet by the space also in feet; and if this product be divided by $16\frac{1}{12}$ times the height of the plane in feet, the square root of the quotient will give the time in seconds.

Example.

Let the length and height of the plane be the same as in the last example, to find how long a body will be in descending down this plane.

Here the length of the plane and the given space are the same; therefore $40 \times 40 = 1600$, then $1600 \div 16\frac{1}{12} \times 1 = 100$ nearly, the square root of which is 10 seconds nearly.

Given the time which a heavy body is descending down an inclined plane, and the velocity acquired, to find the angle of inclination of the plane.

Rule 7.—Divide the acquired velocity by $32\frac{1}{6}$ times the time in seconds, and this quotient will give the sine of the angle of inclination.

Example.

If a body which moves down an inclined plane for 3 seconds acquires a velocity of $48\frac{1}{4}$ feet per second, what is the inclination of the plane?

$$48\tfrac{1}{4} \div 32\tfrac{1}{6} \times 3 = \frac{48\frac{1}{4}}{96\frac{1}{2}} = \tfrac{1}{2}, \text{ which is the sine of } 30^\circ.$$

By formula,

$$sin.\ i = \frac{48\frac{1}{4}}{32\frac{1}{6} \times 3} = \frac{48\frac{1}{4}}{96\frac{1}{2}} = \frac{1}{2}, \text{ the same as before.}$$

Also, this shews that the length of the plane is twice its height, or the accelerating force on the plane is only half the accelerating force of gravity.

To find the angle of inclination when the space described and time are given.

Rule 8.—Divide the space described in feet by $16\frac{1}{12}$ times the square of the time, and the quotient will give the sine of the angle of inclination.

Example.

A body descends $21\frac{4}{9}$ feet from rest along an inclined plane in 2 seconds; required the inclination of the plane.

$21\frac{4}{9} \div 16\frac{1}{12} \times 2^2 = \frac{21\frac{4}{9}}{16\frac{1}{12} \times 4} = \frac{21\frac{4}{9}}{64\frac{1}{3}} = \frac{1}{3}$; that is, the length of the plane is three times its height.

By formula,

$$sin.\ i = \frac{s}{16\frac{1}{12}\ t^2} = \frac{21\frac{4}{9}}{16\frac{1}{12} \times 2^2} = \frac{21\frac{4}{9}}{64\frac{1}{3}} = \frac{1}{3}$$

To find the angle of inclination, when the space described and the velocity acquired are given.

Rule 9.—Divide the square of the acquired velocity by $64\frac{1}{3}$ times the space in feet, and the quotient will give the sine of the angle of inclination.

Example.

A body, by descending 100 feet along an inclined plane, acquires a velocity of $64\frac{1}{3}$ feet; what proportion does the accelerating force on the plane bear to the accelerating force of gravity?

$$(64\tfrac{1}{3})^2 \div 64\tfrac{1}{3} \times 100 = \frac{(64\frac{1}{3})^2}{64\frac{1}{3} \times 100} = \frac{64\frac{1}{3}}{100} = \frac{193}{3 \times 100} = \frac{193}{300}.$$

Note.—The acquired velocity is always given in feet per second.

If the proportion which the length of the plane bears to the height be given, we must substitute these proportions in the foregoing rules.

Thus, suppose that the length of a plane is to its height as 2 to 1, then in these rules use 2 for the length of the plane, and 1 for the height, and the conclusions will be equally as true as those where the length and height are absolutely given.

Example.

In what time will a body descend 30 feet down an inclined plane which rises 1 foot in 10?

Here the length is to the height as 10 to 1; and using 10 for the length and 1 for the height, in Rule 6, we have, $10 \times 30 = 300$, and $300 \div 16\frac{1}{12} \times 1 = 18\cdot65$, the square root of which is $4\cdot32$ seconds nearly.

Or thus, by the formula,

$$t = \sqrt{\frac{l\,s}{16\frac{1}{12}\,h}} = \sqrt{\frac{10 \times 30}{16\frac{1}{12} \times 1}} = 4\cdot32 \text{ seconds.}$$

117. If a body be projected down an inclined plane with a given velocity, then the distance which the body will be from the point of projection in a given time will be $t \times v + \frac{h}{l} \times 16\frac{1}{12}\,t^2$; but if the body be projected upwards, then the distance of the body from the point of projection will be $t \times v - \frac{h}{l} \times 16\frac{1}{12}\,t^2$, retaining the same notation as before.

Example 1.

If a body be projected with a velocity of 40 feet per second, down an inclined plane the length of which is 3 times its height, what space will it move through in 6 seconds?

Here, $t = 6$, $v = 40$, and $\frac{h}{l} = \frac{1}{3}$.

Hence, $t \times v + \frac{h}{l} \times 16\frac{1}{12}\, t^2 = 6 \times 40 + \frac{1}{3} \times 16\frac{1}{12} \times 6^2 = 433$ feet.

Example 2.

If a body be projected upwards with a velocity of 40 feet per second, what distance will it be from the point of projection in 3 seconds?

Here, $t = 3$, $v = 40$, and $\frac{h}{l} = \frac{1}{3}$.

Hence, $t \times v - \frac{h}{l} \times 16\frac{1}{12}\, t^2 = 40 \times 3 - \frac{1}{3} \times 16\frac{1}{12} \times 3^2 = 71\frac{3}{4}$ feet.

Inertia.

118. By inertia is meant the passiveness of matter; that is, matter has not the power of putting itself into motion, neither has it the power of stopping itself when put into motion by the action of an external force, for it requires as much force to stop a body as it requires to put it in motion.

The retardation of motion, arising from inertia, is always proportional to the mass of the body; for the inertia of a body is evidently composed of the inertia of all its parts; therefore, if one body contain twice as much matter as another, it will resist the communication of motion twice as much; or, more properly speaking, the former body will require twice as much force to move it with a given velocity as the latter body will require.

We have already shewn that the momentum of a body is proportional to its velocity and quantity of matter;

consequently, the velocity is proportional to the momentum divided by the quantity of matter.

The force which generates momentum in a body is called a moving force, and the force which generates velocity is called an accelerating force; therefore, if we substitute the moving force, which is proportional to the momentum, for the momentum, and the accelerating force, which is proportional to the velocity, for the velocity, we have f proportional to $\frac{m}{q}$; m being $=$ the moving force, $f =$ the accelerating force, and $q =$ quantity of matter.

If any two bodies P and W be suspended over a pulley moveable about an axis, then, if P be heavier than W, we have $P - W$ for the moving force, but $P + W$ is the mass moved; hence $\frac{P - W}{P + W}$ will represent the accelerating force.

But if the inertia of the pulley be taken into consideration, which call I, then the accelerating force is $\frac{P - W}{P + W + I}$

If, in the formulæ, pages 82 and 83, we substitute $\frac{P - W}{P + W}$ for f, the theorems will apply to this case.

Example 1.

If a weight of 6 *lbs.* act upon a weight of 4 *lbs.* over a pulley, what space will it descend in 6 seconds?

$$\text{Here, } \frac{P - W}{P + W} = \frac{6 - 4}{6 + 4} = \tfrac{2}{10} = \tfrac{1}{5} = f$$

$$s = 16\tfrac{1}{12} f t^2 = 16\tfrac{1}{12} \times \tfrac{1}{5} \times 36 = 115\tfrac{4}{5}.$$

Example 2.

If a weight of 6 *lbs.* be attached to a weight of 2 *lbs.* by means of a cord going over a pulley, how far will the heavier weight descend in 6 seconds?

Here, $P = 6$, $W = 2$, $\therefore \dfrac{P - W}{P + W} = \dfrac{6 - 2}{6 + 2} = \frac{4}{8} = \frac{1}{2}$;
and $s = 16\frac{1}{12} f t^2 = 16\frac{1}{12} \times \frac{1}{2} \times 6^2 = 289\frac{1}{2}$ feet.

Example 3.

Two weights of 1 *lb.* each are suspended by a cord going over a pulley; a weight of 2 *oz.* is added to one of them. How long will it be in descending through 20 feet, and what velocity will it have acquired?

Here, $P = 18$ *oz.* $W = 16$ *oz.* and $s = 20$ feet.

$$\therefore \frac{P - W}{P + W} = \frac{18 - 16}{18 + 16} = \frac{2}{34} = \frac{1}{17} = f.$$

$$t = \sqrt{\frac{s}{16\frac{1}{12} f}} = \sqrt{\frac{20}{16\frac{1}{12} \times \frac{1}{17}}} = \sqrt{\frac{20 \times 17}{16\frac{1}{12}}} = 4{\cdot}6$$
seconds;

and $v = 32\frac{1}{6} f t = 32\frac{1}{6} \times \frac{1}{17} \times 4{\cdot}6 = 8{\cdot}7$ feet per second.

On Rotatory Motion.

119. When two bodies act upon each other as in the last article, their motions are considered to be performed in the direction of a right line passing through their centres of gravity, and therefore each particle of the whole mass is equally accelerated; but if one of the bodies be acted upon by a force so as to produce a rotatory motion in the other body round a fixed axis, then each particle of this body will move with a velocity proportional to its distance from the axis of motion.

120. The Moment of Inertia of any body is the sum of the products of each particle of that body, multiplied by the square of its distance from the axis of motion.

If any system of bodies revolve round a fixed axis, a point may be found, such that if the whole mass of the system were collected in it, the moment of inertia would be the same as before; that is, the sum of the products of

each body, into the square of its distance from the axis of motion, is equal to the sum of all the bodies, multiplied by the square of the distance of this point from the axis of motion.

Or, in a single body, a point may be found, such that if the whole mass of the body were collected in this point, the moment of inertia would be the same as before. This point is called the centre of gyration, and its distance from the axis of rotation is called the radius of gyration.

Or the centre of gyration may be defined to be that point into which, if the whole mass be collected, the same angular velocity will be generated in the same time, by a given force acting at any place, as in the body or system itself.

The angular motion of a body, or system of bodies, is the motion of a line connecting any point and the axis of motion.

If P = the weight or power which acts at the distance r from the axis of motion, to give rotation to a body or system the weight of which is W, and the distance of the centre of gyration from the axis of motion equal $= d$, then the accelerating force

$$f = \frac{P r^2}{P r^2 + W d^2}$$

But if the body or system be put in motion by a power P acting over a pulley, then the inertia of P is nothing.

$$\therefore f = \frac{P r^2}{W d^2}$$

The distance d of the centre of gyration, in some of the most useful cases, is as follows:—

In a circular wheel of uniform thickness.	$d = \frac{1}{2} r \sqrt{2}$
In the circumference of a circle revolving about diameter - - - -	$d = \frac{1}{2} r \sqrt{2}$
In the plane of a circle ditto - - -	$d = \frac{1}{2} r$
In a solid sphere ditto - - - -	$d = r \sqrt{\frac{2}{5}}$

In a circular ring, the radii of which are R and r revolving about the centre $d = \sqrt{\left(\frac{R^2 + r^2}{2}\right)}$*

In a cone revolving about its vertex $d = \frac{1}{2}\sqrt{(\frac{12}{5}a^2 + \frac{3}{5}r^2)}$

In a cone revolving about its axis - $d = r\sqrt{\frac{3}{10}}$

In a straight lever, the arms of which are R and r - - - - - - - $d = \sqrt{\left(\frac{R^3 + r^3}{3(R + r)}\right)}$

In a paraboloid, the radius of the base of which is R - - - - - - $d = R\sqrt{\frac{1}{3}}$

Example 1.

A cylinder, the weight of which is 80 *lbs.* is put in motion by a weight of 20 *lbs.* attached to a string which is coiled round the cylinder. How far will the weight descend in 6 seconds?

$$\text{The accelerating force } f = \frac{P r^2}{P r^2 + W d^2}$$

Here, P = 20 *lbs.* W = 80 *lbs.* and $d = \frac{1}{2} r \sqrt{2}$.

$$\therefore \frac{P r^2}{P r^2 + W d^2} = \frac{P r^2}{P r^2 + W \times \frac{1}{2} r^2} = \frac{P}{P + \frac{1}{2} W}$$

$$= \frac{20}{20 + 40} = \frac{1}{3}.$$

That is, the accelerating force is $\frac{1}{3}$ of the accelerating force of gravity; and by formula, page 82,

$$s = 16\tfrac{1}{12} f t^2 = 16\tfrac{1}{12} \times \tfrac{1}{3} \times 6^2 = 193 \text{ feet.}$$

If the cylinder had been put in motion by a power P without inertia, such as the exertion of animal strength, then, since the inertia of P is nothing, we have the acce-

* In Dr. Gregory's Mathematics for Practical Men, this is given, $\sqrt{\left(\frac{R^4 - r^4}{2R^2 - 2r^2}\right)}$; but it may be reduced so as to agree with the above, as follows:—

$$\sqrt{\left(\frac{R^4 - r^4}{2R^2 - 2r^2}\right)} = \sqrt{\left\{\frac{(R^2 + r^2) \times (R^2 - r^2)}{2(R^2 - r^2)}\right\}}$$

$$= \sqrt{\left(\frac{R^2 + r^2}{2}\right)}$$

lerating force $= \frac{P r^2}{W d^2} = \frac{P r^2}{W \times \frac{1}{2} r^2} = \frac{P}{\frac{1}{2} W} = \frac{20}{40} = \frac{1}{2}$.

$$\therefore s = 16\tfrac{1}{12} \times \tfrac{1}{2} \times 6^2 = 289\tfrac{1}{2} \text{ feet.}$$

Example 2.

If a sphere, the weight of which is 100 *lbs.* and diameter 4 feet, is put in motion by a weight of 20 *lbs.* acting by means of a string going over a pulley 1 foot in diameter, how long will the weight be in descending 100 feet, and what velocity will it have acquired?

Here $P = 20$, $W = 100$, $r = \frac{1}{2}$ a foot; and if the radius of the sphere $= R$, then $d = R\sqrt{\frac{2}{5}}$, or $d^2 = \frac{2}{5} R^2$.

$$\therefore \frac{P r^2}{P r^2 + W d^2} = \frac{P r^2}{P r^2 + W \times \frac{2}{5} R^2} =$$

$$\frac{20 \times \frac{1}{4}}{20 \times \frac{1}{4} + 100 \times \frac{2}{5} \times 4} = \tfrac{5}{165} = \tfrac{1}{33}.$$

$$t = \sqrt{\frac{s}{16\frac{1}{12} f}} = \sqrt{\frac{100}{16\frac{1}{12} \times \frac{1}{33}}} = 14{\cdot}3 \text{ seconds.}$$

$$v = 32\tfrac{1}{6} f t = 32\tfrac{1}{6} \times \tfrac{1}{33} \times 14{\cdot}3 = 13{\cdot}9 \text{ feet per second.}$$

121. The velocity at the circumference of the wheel is always the same as the velocity of the descending weight; and the velocity acquired by the descending weight in the time t is (by the formulæ, pages 82 and 83) $32\frac{1}{6} f t$, which is therefore equal to the velocity at the circumference of the wheel, the radius of which call r; then the velocity at the distance r : the velocity at any other distance x :: $r : x$.

Hence the velocity at the distance x from the axis of rotation is $\frac{32\frac{1}{6} f t x}{r}$ and if $d =$ radius of gyration, then the velocity of the centre of gyration is $\frac{32\frac{1}{6} f t d}{r}$

If the velocity were required in terms of the space, then by the above quoted formulæ, $v = \sqrt{64\frac{1}{3} f s}$, and the velocity at any other distance $x = \frac{x\sqrt{64\frac{1}{3} f s}}{r}$ and the velocity of the centre of gyration $= \frac{d\sqrt{64\frac{1}{3} f s}}{r}$.

And the angular velocity may be found as follows:—

Every point in the circumference of the wheel revolves with a velocity of $32\frac{1}{6} f t$ feet per second; then $32\frac{1}{6} f t$: the whole circumference :: the arc or angle described in one second : 360°.

That is, $32\frac{1}{6} f t : 6.2832\, r$ :: angle described in 1″ : 360°.

$\therefore$ angle described in $1'' = \dfrac{32\frac{1}{6} f t}{6{\cdot}2832\, r} \times 360° = \dfrac{16\frac{1}{12} f t}{3{\cdot}1416\, r} \times 360°$.

That is, the angle described in 1″ by a body or system of bodies revolving uniformly with the velocity acquired, whilst the weight descends for the time t, $= \dfrac{16\frac{1}{12} f t}{3{\cdot}1416\, r} \times 360°$; that is, at the rate of $\dfrac{16\frac{1}{12} f t}{3{\cdot}1416\, r}$ revolutions in a second.

Hence the angular velocity varies as $\dfrac{f t}{r}$; or if t be given, it varies as $\dfrac{f}{r}$*

Also, if a body or system revolve uniformly with the velocity acquired in descending through a given space s, then the number of revolutions which the body or system would make in 1″ is $\dfrac{\sqrt{16\frac{1}{12} f s}}{3{\cdot}1416\, r}$

Example 1.

A fly-wheel, the weight of which is 100 *lbs.* is put in motion by a weight of 20 *lbs.* acting upon a wheel the radius of which is 6 inches; the external and internal radii

* The angular velocity generated in a given time, in any body or system of bodies revolving round a fixed axis, varies directly as the force which generates it, and inversely as the distance from the axis at which the force acts. Or, the angular velocity is as the absolute velocity directly, and the distance from the axis inversely; so that if their absolute velocities be as their radii, the angular velocities will be equal.

of the fly-wheel being 3 and 5 feet respectively. After the weight has descended for the space of one minute, it is disengaged, and the fly-wheel is left to revolve uniformly with the acquired velocity. How many revolutions will it make in a minute, and through what space will the weight have descended?

By page 102, $d^2 = \sqrt{\frac{5^2 + 3^2}{2}} = \sqrt{17} = 4{\cdot}123$ feet.

The accelerating force =

$$\frac{P\,r^2}{P\,r^2 + W\,d^2} = \frac{20 \times \frac{1}{4}}{20 \times \frac{1}{4} + 100 \times 17} = \frac{1}{341} = f.$$

And the number of revolutions which the fly-wheel makes in 1″ =

$$\frac{16\frac{1}{12}\,f\,t}{3{\cdot}1416\,r} = \frac{16\frac{1}{12} \times \frac{1}{341} \times 60}{3{\cdot}1416 \times \frac{1}{2}} = \frac{16\frac{1}{12} \times 60}{3{\cdot}1416 \times \frac{1}{2} \times 341} = 1{\cdot}8,$$

or 108 revolutions in a minute.

The space descended $s = 16\frac{1}{12}\,f\,t^2 = 16\frac{1}{12} \times \frac{1}{341} \times 60^2 = 170$ feet nearly.

Example 2.

A paraboloid, the weight W of which is 200 *lbs.* is put in motion by a weight P of 15 *lbs.* acting upon the wheel A, the radius r of which is 6 inches, and the radius R of the base of the paraboloid is 20 inches. After the weight P has descended for 10″, it is taken off, and the paraboloid is left to revolve uniformly with the velocity acquired at that period. With what velocity does the centre of gyration revolve, and how many revolutions does the paraboloid itself perform in a minute?—*Bridge's Mechanics.*

At page 102, the distance d of the centre of gyration of the paraboloid from the axis is $R\sqrt{\frac{1}{3}} = 20\sqrt{\frac{1}{3}}$.

The accelerating force of the weight $P = \frac{P\,r^2}{P\,r^2 + W\,d^2}$

$= \frac{15 \times 36}{15 \times 36 + 200 \times \frac{1}{3} \times 400} = \frac{1}{50{\cdot}3} = f$; and by Art. 121,

the velocity of the centre of gyration $= \frac{32\frac{1}{6}\,f\,t\,d}{r} =$

$$\frac{32\frac{1}{6} \times 10 \times 20 \sqrt{\frac{1}{3}}}{6 \times 50{\cdot}3} = 12{\cdot}1 \text{ feet in a second.}$$

Also, by Art. 121, the number of revolutions which the paraboloid makes in $1'' = \frac{16\frac{1}{12} f t}{3{\cdot}1416\, r} = \frac{16\frac{1}{12} \times 10}{3{\cdot}1416 \times \frac{1}{2} \times 50{\cdot}3}$ $= 2{\cdot}03$, or 121 revolutions per minute.

ON PENDULUMS.

122. A Pendulum is either simple or compound. A simple pendulum consists of a particle of matter fastened to the end of a very fine inextensible string, the other end being fastened to a pin, about which it vibrates as a centre of motion. A compound pendulum consists of two or more bodies, or of one body from the figure and extent of which we are not permitted to abstract.

123. The centre of oscillation of a compound pendulum is a point in it at such a distance from the centre of suspension, that a simple pendulum, of a length equal to that distance, will have the same angular velocity with the compound pendulum itself.

124. The centre of percussion, which is generally in the same point as the centre of oscillation, may be explained as follows:—

In striking any body with a bar or lever, it is always found that if the blow is given at or near the end of the bar, it will jar, or attempt to fly out of the hand; and if the blow is given by that part of the bar near the hand, it will also jar, and attempt to fly from it. Now there

evidently must be a point between these two, where, if a stroke is given, the full effect of the blow will be sensible, and the bar will remain at rest, without jarring the hand. This point is called the centre of percussion, or the point in a striking body where, if it strike another, the effect will be most powerful; and as the centre of gravity of a body is a point on which, if suspended, the body would be in equilibrio, so the centre of percussion is a point in which the whole momentum of the moving body is placed to produce the greatest effect.

On the Simple Pendulum.

125. It has been found, by many very accurate experiments, that a pendulum which vibrates seconds in the latitude of London is $39\frac{1}{8}$ inches in length. This being known, we can find the length of a pendulum which will make any number of vibrations in a given time, as follows:

Bring the given time into seconds; then, as the square of the number of vibrations given is to the square of the given number of seconds, so is $39\frac{1}{8}$ to the length of the required pendulum in inches.

Example.

What must be the length of a pendulum, so as to make 80 vibrations in a minute?

Here the given time is 60 seconds.

$$6400 : 3600 :: 39\tfrac{1}{8} : \frac{39\frac{1}{8} \times 3600}{6400} = \frac{140850}{6400} = 22 \text{ inches.}$$

Therefore, if the length of a pendulum be required, so as to make a given number of vibrations in a minute, divide 140850 by the square of the number of vibrations given, and the quotient will be the length of the pendulum.

Example.

What must be the length of a pendulum to make 50 vibrations in a minute?

$$\frac{140850}{2500} = 56{\cdot}34 \text{ inches.}$$

Given the length of a pendulum, to find how many vibrations it will make in a given time.

Bring the given time into seconds; then, as the given length of the pendulum is to $39\frac{1}{8}$, so is the square of the given time to the square of the number of vibrations, the square root of which is the number sought.

Example.

If the length of a pendulum be 48 inches, how many vibrations will it make in a minute?

The given time is 60 seconds.

$$48 : 39{\cdot}125 :: 60^2 : \frac{39{\cdot}125 \times 3600}{48} = \frac{140850}{48} = 293{,}438,$$

the square root of which is 54·17 vibrations in a minute.

Centre of Oscillation and Percussion.

126. The distance of the centre of oscillation or percussion of any compound pendulum from its centre of suspension, is equal to the sum of the products of each body into the square of its distance from the centre of suspension, divided by the sum of the products of each body into its distance from that centre.

Thus, if any number of bodies, A, B, C, &c. and their respective distances from the centre of suspension, a, b, c, &c. be given, then the distance of the centre of oscillation from the centre of suspension is $\dfrac{A\,a^2 + B\,b^2 + C\,c^2}{A\,a + B\,b + C\,c}$

Let A = 4 *lbs.* and its distance from the centre of suspension 4 inches, B = 6 *lbs.* and its distance 2 inches, and C = 8 *lbs.* and its distance from the same point 3 inches; then, $\dfrac{4 \times 4^2 + 6 \times 2^2 + 8 \times 3^2}{4 \times 4 + 6 \times 2 + 8 \times 3} = \dfrac{64 + 24 + 72}{16 + 12 + 24} =$ $3\frac{1}{13}$; that is, the centre of oscillation is $3\frac{1}{13}$ inches from the centre of suspension.

The distance of the centres of oscillation and percussion from the axis of motion is as follows, where the axis of motion is at the vertex and in the plane of the figure:—

In a right line, small parallelogram, and cylinder, $\frac{2}{3}$ the axis of the figure.

In a triangle, $\frac{3}{4}$ the axis.

In the parabola, $\frac{5}{7}$ of the axis.

If a cylinder, of which the altitude is a, and the radius r, be suspended from its vertex, the distance of the centre of oscillation from the vertex is $\frac{2a}{3} + \frac{r}{2a}$

If a cone be suspended from the vertex, the altitude of which is a, and the radius of the base r, the distance of the centre of oscillation from the vertex is $\frac{4a}{5} + \frac{r^2}{5a}$

In a sphere, $r =$ radius, $d =$ distance of the axis of motion from its centre; then the distance of the centre of oscillation from the axis of motion is $d + \frac{2r^2}{5d}$

If the sphere be suspended by a point in its surface, then the distance of the centre of oscillation from that point is $\frac{7r}{5}$

Example 1.

If a cylinder, of which the radius is 4 inches and altitude 1 foot, be suspended by its vertex; required the length of a simple pendulum which will vibrate in the same time.

Here, $r = 4$ inches, $a = 12$.

$$\therefore \frac{2a}{3} + \frac{r}{2a} = \frac{2 \times 12}{3} + \frac{4}{2 \times 12} = 8\tfrac{1}{6} \text{ inches.}$$

Example 2.

If a globe, the radius of which is 6 inches, be suspended by a point in its surface, required the length of a simple pendulum which will vibrate in the same time.

Here $r = 6$ inches, and $\frac{7\,r}{5} = \frac{7 \times 6}{5} = 8\frac{2}{5}$ inches.

If the globe be suspended by a string 8 inches long, attached to a point in its surface, then the length of the pendulum is $d + \frac{2\,r^2}{5\,d}$

Since $r = 6$, we have $d = 8 + 6 = 14$.

$$\therefore d + \frac{2\,r^2}{5\,d} = 14 + \frac{2 \times 6^2}{5 \times 14} = 15\tfrac{1}{35} \text{ inches.}$$

CENTRAL FORCES.

127. Centripetal force is that force or power which tends constantly to impel bodies towards a fixed point or centre.

128. Centrifugal force is that by which bodies would recede from such a centre, were they not prevented by the centripetal force.

129. These two forces are jointly called Central Forces.

130. When a body describes a circle by means of a force directed to its centre, its actual velocity is every where equal to that which it would acquire in falling by the same uniform force through half the radius.

131. This velocity is the same as that which a second body would acquire by falling through half the radius, whilst the first describes a portion of the circumference equal to the whole radius.

132. In equal circles, the forces are inversely as the squares of the times.

133. When the times are equal, the velocities are as the radii, and the forces are also as the radii.

134. And, generally, the forces are as the distances or radii of the circles directly, and the squares of the times inversely.

135. The squares of the times are as the distances directly and the forces inversely.

136. Therefore if the forces are inversely as the square of the distances, the squares of the times are as the cubes of the distances.

137. The centrifugal force of a body revolving in a circle is equal to the square of the velocity divided by the radius.

Let F and f represent the forces, T and t the times, and D and d the distances; then,

$$F : f :: t^2 : T^2 \qquad F : f :: \frac{D}{T^2} : \frac{d}{t^2}$$

$$F : f :: \frac{1}{T^2} : \frac{1}{t^2} \qquad T^2 : t^2 :: \frac{D}{F} : \frac{d}{f}$$

Example 1.

Suppose the diameter of a grindstone to be 44 inches, and its weight half a ton; suppose also that it makes 326 revolutions in a minute; what will be the centrifugal force, or its tendency to burst?

The radius of the circle of gyration is $\frac{r}{2}\sqrt{2}$, or the diameter $= r\sqrt{2} = 22\sqrt{2} = 31{\cdot}11$ inches $= 2{\cdot}59$ feet.

d being the radius of gyration,

$$\frac{2\,d\,(3{\cdot}1416)^2}{16\frac{1}{12}\,t^2} = \frac{2{\cdot}59 \times (3{\cdot}1416)^2}{16\frac{1}{12} \times (\frac{60}{326})^2} = 47{\cdot}18 \text{ times the}$$

weight of the stone; hence,

$$\frac{47{\cdot}18}{2} = 23{\cdot}59 \text{ tons, the centrifugal force.}$$

Example 2.

If a fly-wheel, 10 feet diameter and 2 tons weight, perform a revolution in 6 seconds, and suppose another of the same weight revolves in 4 seconds, what must be the dia-

meter of this last, so that their centrifugal forces may be equal?

By Art. 137, $F : f :: \frac{D}{T^2} : \frac{d}{t^2}$; but, in this case, the centrifugal forces are equal, that is $F = f$; hence, $\frac{D}{T^2} = \frac{d}{t^2}$

$$\therefore d = \frac{D\,t^2}{T^2} = \frac{10 \times 4^2}{6^2} = \frac{10 \times 16}{36} = \frac{160}{36} = 4\tfrac{4}{9} \text{ feet.}$$

Example 3.

If a fly-wheel, 10 feet diameter, revolve in 6 seconds, and another of the same diameter in 4 seconds, what proportion do the weights bear to each other, when their central forces are equal?

The weights are as the square of the times; that is,

$$w : W :: 16 : 36 :: 1 : 2\tfrac{1}{4}.$$

Example 4.

If a fly, 2 tons weight and 16 feet in diameter, is sufficient to regulate an engine when it revolves in 4 seconds, what must be the weight of another fly, of 12 feet diameter, revolving in 2 seconds, so that it may have the same power upon the engine?

For each fly to have the same power on the engine, their centrifugal forces must be equal.

$F : f :: \frac{W\,D}{T^2} : \frac{w\,d}{t^2}$; but $F = f$; hence, $\frac{W\,D}{T^2} = \frac{w\,d}{t^2}$

$\therefore w = \frac{W\,D\,t^2}{d\,T^2} = \frac{40\ cwt. \times 16 \times 2^2}{12 \times 4^2} = \frac{160}{12} = 13\tfrac{1}{3}$ *cwt.* the weight of the smaller fly.

On the Conical Pendulum, or Governor.

138. Let A P and A Q represent the arms of the governor, P and Q the balls; and let x = B P, the variable horizontal distance of each ball from that shaft, a = the

corresponding altitude, $t =$ the time of one revolution of the shaft, and $p = 3{\cdot}1416$. (See Fig. 1 to Steam Engine.)

Then the velocity of each ball $= \frac{2\,p\,x}{t}$; and since the centrifugal force is equal to the square of the velocity divided by the radius, we have $\left(\frac{2\,p\,x}{t}\right)^2 \div x = \frac{4\,p^2\,x}{t^2}$

The balls are acted on by two forces, the centrifugal force and gravity; the former acting in a direction parallel to the horizon, and the latter in a direction perpendicular to it. The resultant of these two forces is evidently always in the direction of the arms A P or A Q. It follows, therefore, that the ratio of the gravity to the centrifugal force is as A B to B P.

$$a : x :: g : \frac{4\,p^2\,x}{t^2}$$

$$\therefore g\,t^2 = 4\,a\,p^2$$

$$t = 2\,p\,\sqrt{\frac{a}{g}} = 1{\cdot}10784\,\sqrt{a}$$

Hence the periodic time varies as the square root of the altitude of the conical pendulum, let the radius of the base be what it may.

When A B = B P, the centrifugal force of each ball is equal to its weight.

PART II.

ON THE STEAM ENGINE.

THE Steam Engine is, beyond all doubt, one of the noblest inventions which any age or country can boast of; and many disputes have arisen concerning its origin. Some have considered the famous Denys Pepin as its inventor; others ascribe it to the Marquis of Worcester. But be this as it may, we are quite certain that it is by the great skill, powerful genius, and unremitting assiduity of the celebrated Watt, that it has attained its present state of usefulness and perfection.

Fuel, &c.

Mr. Watt found that, with the most judiciously constructed furnaces, it required 8 feet of surface of the boiler to be exposed to the action of the fire and flame to boil off a cubic foot of water in an hour; and that a bushel of Newcastle coals, so applied, will boil off from 8 to 12 cubic feet; and that it requires about a *cwt.* of Wednesbury coals to do the same.

A bushel of coals, which is the consumption of a 10 horse engine for one hour, grinds and dresses about 10 bushels of wheat, Winchester measure.

The consumption of a bushel of good Newcastle coals (= 84 *lbs.*) will raise 18·34 millions pounds of water one

foot high. Or 1·08 bushels per hour will supply an engine of 10 horse power. And, at the same rate, each horse power will require 9·07 *lbs.* of such coals per hour.

For a bushel of coals (= 84 *lbs.*) may be assumed to evaporate 10·08 cubic feet of water into steam; and every cubic inch of water, so evaporated, will produce a cubic foot of steam, of the same elasticity as the atmospheric air. Hence, a bushel of coals will produce $(10{\cdot}08 \times 1728) =$ 17418 cubic feet of steam; and if each cubic foot of steam raises one cubic foot of water 16·86 feet high, then the consumption of a bushel of coals must raise $17418 \times 16{\cdot}85$ = 293500 cubic feet of water, or 18340000 *lbs.* of water, one foot high.*

When bodies are heated above the temperature of the surrounding media, the heat escapes from them by radiation; that is, by passing from the body in straight lines through that surrounding media.

Professor Leslie has proved, by a variety of experiments, that the heat which is propagated by radiation from different bodies, varies with the nature of their external surfaces; the quantity which flows in a given time from a body with a polished surface being much less than would flow from the same body with a rough surface. It, therefore, follows, that the external surfaces of the steam pipes of steam engines and steam cylinders should be as smooth as possible, and should be covered with any body which is a bad conductor of heat.†

Dr. Hook found that if water were put into any vessel and set on the fire, the temperature of the water would continually increase until it began to boil; after which, its temperature would increase no further, as much heat being carried off by the vapour as is received from the fire. But Papin, by using enclosed vessels, found that the tem-

* Farey on the Steam Engine, page 488.

† Experimental Enquiry into the Nature and Propagation of Heat, page 17.

perature of water could be raised to a very great height; and Muschenbroeck informs us that he has raised water to a temperature which would melt tin.

STEAM, &c.

An elaborate set of experiments have appeared, by a a committee of the Royal Academy of Sciences of Paris, consisting of Prony, Arago, Gerard, and Dulong. These experiments were the result of an application of the French government to the Academy, to present the best means of preventing accidents from the bursting of the boilers of steam engines.

The following table exhibits the elasticity of steam at various temperatures, until it amounts to 24 atmospheres.

An atmosphere is measured by a column of mercury of 29·922 inches, which has been adopted in France as the mean height of the barometer at the surface of the sea.

Elasticity of steam, the pressure of the atmosphere being 1.	Corresponding temperature in degrees of Fahrenheit.	Elasticity of steam, the pressure of the atmosphere being 1.	Corresponding temperature in degrees of Fahrenheit.
1	212°	10	358·88
1½	234	11	366·85
2	250·5	12	374·
2½	263·8	13	380·66
3	275·2	14	386·94
3½	285·	15	392·86
4	293·7	16	398·48
4½	300·3	17	403·83
5	307·5	18	408·92
5½	314·24	19	413·78
6	320·36	20	418·46
6½	326·26	21	422·96
7	331·7	22	427·28
7½	336·86	23	431·42
8	341·78	24	435·56
9	350·78		

It is a very curious circumstance, that low pressure steam will scald most dreadfully, while high pressure steam produces no such effect. This curious fact is explained as follows by Dr. Thompson*:—

"When the steam of boiling water comes in contact with any part of the living body, it occasions a most severe scald; but when steam from water of a higher temperature than boiling water, or high pressure steam as it is called, issues into the atmosphere, the finger or any part of the body may be passed through it with impunity. It has not the property of scalding. And if a thermometer be put into it, we find the temperature greatly below that of boiling water; so that high pressure steam has a much lower temperature than low pressure steam, or steam issuing freely from boiling water.

"Whoever has an opportunity of seeing these two different species of steam, will find no difficulty in understanding the reason of this difference. When steam issues from the spout of a boiling tea-kettle, it is at first invisible; and it is not till it has advanced some distance in the air, that it begins to assume the appearance of a visible cloud. But condensed steam is visible the instant that it issues from the mouth of the pipe. The high pressure steam (supposing its elasticity double) occupies only half the bulk of common steam. The moment it comes into the atmosphere, its volume is doubled. This occasions a prodigious increase in its capacity for heat, and at the same time mixes it with the cold atmospheric air. These two circumstances sink its temperature so low that it is no longer capable of scalding."

On Boilers.

Mr. Millington considers that a boiler for 20 horse power is usually 15 feet long and 6 wide, therefore 90 feet

* Outlines of Heat and Electricity, page 209.

considered; for when the lever is large and the valve is small, the weight of the lever is such as to produce a very sensible pressure upon the valve to each square inch. But previous to the calculation, it will be necessary to make the following remarks.

Since the fulcrum is at one end, and the power or the action of the steam between that end and the moveable weight (see Fig. 2, where F is the fulcrum, A is the point where the steam acts, and W the moveable weight), some have taken A for the fulcrum, and thereby have committed very great errors; for, according to this rule, a weight put on at twice the distance from A that A is from F, would, if the weight of the lever were not considered, be twice its own weight upon the valve; whereas, if it had been reckoned from F, its real fulcrum, it would be three times its own weight upon the valve.

It has been shewn, and indeed it is almost self-evident, that if we have two, three, or four times, &c. the leverage, we will have two, three, or four times, &c. the effect produced respectively, the weight remaining the same.— Therefore divide the length of the lever by the distance between the fulcrum and valve, and the quotient gives the leverage; and the leverage, multiplied by the weight, gives the whole weight upon the valve; and this product, divided by the number of square inches in the valve, gives the weight per square inch. Or, if the weight per inch be known, multiply the number of pounds per square inch by the number of square inches, and this product gives the whole weight upon the valve, which, divided by the leverage, gives the weight. Or, if the weight be given, divide by it, and the quotient will give the leverage; and the leverage, multiplied by the distance between the fulcrum and the valve, gives the length of the lever.

Ex.—Given the whole length of the lever 24 inches, the distance between the fulcrum and valve 3 inches, the diameter of the valve 2½ inches; required the weight put on at the end of the lever, so as to have 50 *lbs.* per square

inch upon the valve; also to divide the lever so as to have 40, 30, 20 *lbs.* &c. upon the valve with the same weight.

$(2\cdot5)^2 \times \cdot7854 = 4\cdot9 =$ area of the valve.

$4\cdot9 \times 50 = 245$ *lbs.* whole weight on the valve.

$\frac{245}{8} = 30\cdot625$ *lbs.* = the weight which must be put on at the end of the lever to give 50 *lbs.* per square inch.

And $\frac{4\cdot9 \times 40}{30\cdot625} = 6\cdot4$; then, $6\cdot4 \times 3 = 19\cdot2$ inches, the distance from the fulcrum the weight must be placed to have 40 *lbs.*

$24 - 19\cdot2 = 4\cdot8$; that is, the weight must be shifted in towards the fulcrum 4·8 inches to have 40 *lbs.* per inch; and for 30 *lbs.* per square inch, move it in 4·8 inches more, &c.

To find what weight must be put on at the end of a lever to give any number of pounds pressure per square inch upon the valve, the weight of the lever being taken into consideration.

Rule.—Find the area of the valve by multiplying the square of the diameter by ·7854; then multiply this area by the number of pounds per square inch which you want upon the valve, and this product will give the whole weight upon the valve.

Next divide the whole length of the lever by the distance between the fulcrum and valve,* and the quotient will give the leverage which any weight will have when put on at the end of the lever.

Multiply this leverage by half the weight of the lever, and the product will give the pressure on the whole valve from the action of the lever alone: add to this product the weight of the valve, &c. and subtract the sum from the

* What is here meant by the distance between the fulcrum and valve, is that part of the lever between the fulcrum and the point where the lever acts upon the valve.

whole weight on the valve above mentioned; the remainder will give the weight which will be pressing on the valve from the action of the weight alone; and this, divided by the leverage, gives the weight itself.

Note.—If, instead of considering half the weight of the lever to act at the end, we conceive the whole weight to act at the centre of gravity, the result will be the same, the lever being uniform.

Ex. 1.—Given the length of the lever 24 inches, the distance between the fulcrum and valve 3 inches, the weight of the valve 3 *lbs.* the weight of the lever 3 *lbs.*; it is required to determine what weight must be put upon the end of the lever that it may press 30 *lbs.* per square inch, the diameter of the valve being 3 inches.

Now, $3 \times 3 = 9$, and $9 \times \cdot 7854 = 7\cdot 0686 =$ area, or number of square inches in the valve; and $7 \times 30 = 210$ *lbs.* = whole weight upon the valve; and if we conceive the whole weight of the lever to be concentrated in its centre of gravity, and acting with the leverage of the centre of gravity, the lever being uniform throughout its length, we have, since $12 =$ the distance between the fulcrum and centre of gravity, $12 \div 3 = 4$, the leverage of the centre of gravity; and 3 *lbs.* the weight of the lever, multiplied by 4, gives 12 *lbs.* the weight that the lever will give upon the whole valve. 12 *lbs.* added to 3 *lbs.* the weight of the valve, gives 15 *lbs.* the weight from both lever and valve; and this, subtracted from 210 *lbs.* gives 195 *lbs.* the weight upon the valve from the action of the weight alone, independent of the weight of the lever; and this, divided by the leverage, gives the weight. Thus, $24 \div 3 = 8 =$ leverage of the end of the lever; and $195 \div 8$ gives $24\frac{3}{8} = 24\cdot 375$ *lbs.* the weight put on at the end of the lever to give 30 *lbs.* per inch, when the weight of the lever is taken into consideration.

Now, this being determined, we must mark the lever in those points, where the above found weight will give 20 *lbs.* per inch, and also 10 *lbs.* per inch. This is found by

inverting the above operation; for you have the weight given, the valve, &c. to find the leverage; and the leverage, multiplied by the distance between the fulcrum and valve, gives the distance from the fulcrum the given weight must be put. Thus $7 \times 20 = 140 =$ whole weight upon the valve; from this subtract 15 *lbs.* the weight from the valve and lever, and the remainder gives what the weight must put on $= 125$ *lbs.*; and this weight, divided by the given weight, 24.375 *lbs.* gives $5{\cdot}128 =$ the leverage; and $5{\cdot}128 \times 3 = 15{\cdot}384$ inches from the fulcrum, for 20 *lbs.* per inch.

Now, to determine the distance from the fulcrum when there are 10 *lbs.* per inch, proceed as above. Thus, $7 \times 10 = 70$ *lbs.* upon the whole valve; subtract from this again 15 *lbs.* the weight of the lever and valve, and 55 remains; and $55 \div 24{\cdot}375 = 2{\cdot}256 =$ leverage; and $2{\cdot}256 \times 3 = 6{\cdot}768$ inches from the fulcrum.

Ex. 2.—Given the length of the lever 16 inches, and its weight 2 *lbs.*; the distance between the fulcrum and valve 2 inches, and the weight of the valve and spindle $1\frac{1}{2}$ *lb.*; to find what weight must be put on at the end of the lever to press 40 *lbs.* per square inch upon the valve, the diameter of which is 2 inches.

The square of 2 is 4; hence,

$$\begin{array}{r} {\cdot}7854 \\ 4 \\ \hline 3{\cdot}1416 \end{array}$$

$3{\cdot}1416 =$ area of the valve, or number of sq. inches in it.

40 *lbs.*

$125{\cdot}6640$ *lbs.* $=$ weight on the whole valve.

$16 \div 2 = 8 =$ leverage which the weight will have at the end.

Now, to consider half the weight of the lever to act at the end, is the same as to consider the whole weight of the lever to act at its centre of gravity, the lever being uniform

1 *lb.* = half the weight of the lever.
8 = leverage at the end of the lever.

8 *lbs.* = weight on the whole valve from the action of the lever.
1·5 *lbs.* = weight of valve, &c.

9·5 *lbs.* = weight on the valve from the action of both lever and valve.

125.664
9·5

116·164 *lbs.*

That is, the weight put on at the end of the lever must be such as to press 116·164 *lbs.* on the whole valve; but the leverage of the weight is 8, therefore one-eighth part of this weight will do. Thus, 116·164 ÷ 8 = 14·52 *lbs.* = weight which must be put on at the end to press 40 *lbs.* per inch upon the valve. Now, to mark the lever where we will have 35 *lbs.* 30 *lbs.* 25 *lbs.* 20 *lbs.* &c. per square inch, we must proceed thus:—

3·1416 = number of square inches in the area of the valve.
35

157080
94248

109·956
9·5

100·456 = weight on the valve from the action of the weight.

And 100·456 ÷ 14.52 = 6·918 = the leverage which the weight must have; and if this leverage be multiplied by the distance between the fulcrum and valve, thus,—

6·918 = leverage.
2 = distance between the fulcrum and valve.

13.836 = the distance along the lever from the fulcrum.

16 — 13·836 = 2·164 inches = the distance which the weight must be moved in towards the valve.

And if you want 30 *lbs.* per square inch, move it in 2·164 inches farther; and so on, as far as you please, making the division always 2·164 inches.

On Parallel Motion.

The ingenious and justly celebrated Mr. Watt, whose inventive genius shines forth in every department of mechanical science, was also the first who invented Parallel Motion, to make the piston rod move nearly in a vertical right line. The principle he describes nearly as follows, in the Appendix which he wrote to the article Steam Engine, in Dr. Robison's Mechanical Philosophy.

Suppose A B and C D (Fig. 3) to be two rigid and inflexible bars or beams, connected by an inflexible link B D; when the beams are horizontal, the link is perpendicular to them; the beams vibrate vertically about the points A and C, and the link moves on the joints at B and D. Now, if the beams are equal, the middle point P of the link will describe a line differing very little from a vertical straight line, providing the beams do not vibrate through an arc much greater than 40 degrees; and if the beams are unequal, suppose in the ratio of one to two, the parts of the link will be in the same proportion; only observing that the longer part of the link must be next the end of the shorter beam. Thus, let A B be the shortest beam, and C D the longest; then, $AB : CD :: DP : BP$, or $AB + CD : AB :: DP + BP (= BD) : DP$. Hence, $DP = \frac{AB \times BD}{AB + CD}$; and, in a similar manner, we have $BP = \frac{BD \times CD}{AB + CD}$. This has also been investigated in the Newcastle Magazine for July, 1821, and the same results deduced, by the late excellent mathematician, Mr. Henry Atkinson, of Newcastle upon Tyne.

Ex. 1.—Given the length of the beam A B = 6 feet, the length of the link B D = 4 feet, and the piston rod is

attached to a point P, which is 2 feet 6 inches from the top of the link B; required the length of the other beam C D.

Since the whole length of the link is 48 inches, and B P = 30 inches, D P will be 18 inches.

D P : B P :: A B : C D; that is, 18 : 30 :: 72

$$\begin{array}{r} 72 \\ 30 \\ \hline 18)2160 \\ \hline 120 \text{ inches,} \end{array}$$

or 10 feet, the length of the beam C D.

Ex. 2.—Given the length of the beam A B = 6 feet, the length of the beam C D = 10 feet, and the length of the link = 4 feet, to find that part of the link where the piston rod must be attached, that it may describe a line differing insensibly from a vertical straight line.

Practically:

Divide the link into two parts, having the same proportion to each other as the beams, observing that the greatest part must be next to the shortest beam.

By calculation:

$$B\,P = \frac{B\,D \times C\,D}{A\,B + C\,D} = \frac{4 \times 10}{6 + 10} = \frac{40}{16} = 2 \text{ feet } 6 \text{ inches.}$$

Hence, D P = 48 — 30 = 18 inches = 1 foot 6 inches.

We will now proceed to shew the application of this principle to the parallel motion described in Fig. 4.

Join A and C, cutting B D in F. Now, since C′ C and B D are always parallel in every position that the beam A C′ takes during the whole length of the stroke, the triangle A B F and A C′ C are always similar; consequently, A B : A C′ :: A F : A C; and since the ratio of A B to A C′ is constant, the ratio of A F to A C is constant. Therefore, whatever kind of a line is described by the point F, the very same kind of line must be described by the point C; but the latter will be longer than the former in the ratio of A F to A C. But, by the last proposition,

D F : F B :: A B : D P; but D C = B C′, and the triangles F D C and F B A are similar. Therefore, F D : F B :: D C : A B :: B C′ : A B; consequently, B C′ : A B :: A B : D P. Hence, $D P = \frac{A B^2}{B C'}$; that is, the radius is a third proportional to B C and A B.

Rule.—Divide the square of the distance between A and B by the distance between B and C′, and the quotient is the length of the radius rod.

But it is sometimes more convenient to attach the radius rod to some other point E or *e*, either above or below the point D. (See Fig. 5.) In these cases, we have only to find the length of the radius rod necessary to make the point F move in a vertical line; for, whatever kind of a line is described by the point F, the very same kind of line will be described by the point C. And when the length of the beam A C′, the length of the link C′ C, and the length of the parallel bar D C, are all given, the point F is also given; for A C′ : C′ C :: A B : B F.

Ex. 1.—Given the length of the beam A C′ = 10 feet, the length of the link C′ C = 5 feet, and the length of the parallel bar = 4 feet, to find the length of the radius rod, when it is attached to a point E, 4 feet 6 inches from B.

Here, A C′ : C′ C :: A B : B F; that is, 10 ft. : 5 ft. :: 6 ft. : 3 ft. = B F.

And B E — B F = E F = 4·5 — 3 = 1·5.

Then, E F : B F :: A B : P E; that is, 1·5 ft. : 3 ft. :: 6 ft. : 12 ft. = P E, the length of the radius rod required.

Ex. 2.—Given, as in the last example, the length of the beam 10 feet, the length of the link 5 feet, and the length of the parallel bar 4 feet, to find the length of the radius rod, when it is attached to a point *e*, 6 feet from B, the top of the back link.

A C′ : C′ C :: A B : B F. 10 : 5 :: 6 : 3 = B F.

And B *e* — B F = F *e* = 6 — 3 = 3. Now, since B F and F *e* are each equal 3, the radius rod will be equal

A B = 6 feet; for, when the parts of the link are equal, the beams are also equal.

Referring to Fig. 3, since G H = B D + L E — E O, and G′ H′ = G′ I + E L* — B D, it follows that G H is always greater than G′ H′; or the beam C D is never parallel to the horizon when it has described half its length of stroke, supposing the other beam to move equally above and below the horizontal position A B; for B D + E L is always greater than G I + E L, and E O less than B D.

These rules are very well adapted for steam-packet engines; for these engines generally have very short strokes, and their links are in most cases very long.

We will here give a few examples, which will sufficiently elucidate these rules.

If, in Fig. 6, the radius rod be put on at D, it is just the same as the common parallel motion turned upside down; therefore $A B^2 + B C$ gives the length of the radius rod. But it is sometimes more convenient to attach the radius rod to some other point E, between D and F; therefore, to make it as general as possible, let E be any given point whatever between D and F; and when the length of the beam A C′, and the length of the link C′ C, are both given, the point F is given; for A C′ : C′ C :: A B : B F. Or it may be done practically, by stretching a string or chalk line between A and C, and it will cut B D in the point F.

Ex. 1.—Given the length of the beam A C′ = 5 feet, the length of the link or side rod C′ C = 4 feet 2 inches, and the distance A B of the back link from the centre of motion 3 feet; it is required to determine the length of the radius rod, when it is attached to a point E, 4 feet above B.

Now, A C′ : C′ C :: A B : B F.

That is, 60 : 50 :: 36 : 30 = B F.

B E — B F = E F = 48 — 30 = 18.

* E L = L F.

E F : F B :: A B : D E.
18 : 30 :: 36
30

18)1080

60 inches = 5 feet = D′ E, the length of the radius rod required.

Ex. 2.—Given the length of the beam A C′ = 5 feet, the length of the links = 4 feet 2 inches, and the distance A B = 2 feet, to find the length of the radius rod, when it is attached to a point E, 4 feet above the point B.

60 : 50 :: 24
50

6,0)120,0

20 inches = B F.

And B E — B F = F E = 48 — 20 = 28.

28 : 20 :: 24
20

28)480

$17\frac{1}{7}$ inches = D′ E, the length of the radius rod.

Ex. 3.—Let the length of the beam A C′ be given, as in the last examples, = 5 feet, and A B = 2 feet, but the length of the radius rod is confined to be only 8 inches; it is required to determine the distance of the point E from B, where the radius rod must be attached, the length of the links being the same as in the last two examples.

A C′ : C′ C :: A B : B F.
60 : 50 :: 24
50

6,0)120,0

20 = B F.

$$D'E : AB :: BF : FE.$$

$$8 : 24 :: 20$$

$$\frac{\begin{array}{r} 20 \end{array}}{8)480}$$

$$\overline{60} = 5 \text{ feet} = EF.$$

And $60 + 20 = 80 = 6$ feet 8 inches $=$ B E, the length of the back link.

To construct a Parallel Motion geometrically.

If the distance of the end C of the beam or parallel bar (according as the engine works with a vibrating pillar or parallel motion) from the point D, where the radius rod is attached to the beam or end of parallel bar, and the length of the stroke, be given, to find the length of the radius rod:

Let C H be the vertical line, and A C, *a* E, and A H, represent the beam or parallel bar, produced at the top extremity, at half stroke, and at the bottom extremity of the stroke respectively.

Take the given distance C D in your compasses, and from E set off E P = C D; also from H apply H G also equal to C D. Then, through the three points D, P, and G, describe a circle, the radius of which is the length of the radius rod required.

From when the beam or parallel bar comes into the position *a* E at half stroke, the point D must coincide with P, so that C may be on the vertical line at E, and coincide with it; and when the point D arrives at G, the point C coincides with H, which is on the vertical line.

Calculation, derived from this construction, is as follows:

Draw D I parallel to A E, and D O to C H; also join D and P. Now, since A C and C D are both given, their difference A D is given. Then, since the triangles A C E and A D O are similar, we have A C : C E :: A D : D O. Hence, D O is given; and since D I is parallel to A E, D O = E I, and C E — D O = C I, and $\sqrt{}$(D C^2 —

$C I^2) = D I = O E$. But $C D = P E$, by construction; therefore, $P E - O E = P O$, the versed sine of the arc described by the radius rod; and $P O^2 + O D^2 = P D^2$; and by Euclid, Cor. to Prop. 8, Book 6, we have $P D^2$ divided by $2 P O =$ the length of the radius rod.

To illustrate this by an example,—Let the length of the beam be 6 feet, length of the stroke 4 feet, and length of the parallel bar 2 feet, to find the length of the radius rod.

Now, since $C H = 4$ feet, $C E = 2$ feet; and, by the above, $6 : 2 :: 4 : \frac{4}{3} = D O$, and $2 - \frac{4}{3} = \frac{2}{3} = C I$; also, $\sqrt{(4 - \frac{4}{9})} = \sqrt{\frac{32}{9}} = 1{\cdot}8855 = D I = O E$, and $2 - 1{\cdot}8855 = {\cdot}1145$. Then, $({\cdot}1145)^2 + (\frac{4}{3})^2 = 1{\cdot}79088802 = D P^2$; and $1{\cdot}79088802$, divided by ${\cdot}1145 \times 2$, gives $7{\cdot}8$ feet, the length of the radius rod.

We will now give a general expression for the length of the radius bar, when the point p is given, the length of the stroke, and the length of the beam.

Let $A C = a$, $C E = b$, $E p = c$, $A E = d$, and $D C = x$; then, by similar triangles, $A C : C E :: D C : C I$; that is, $a : b :: x : \dfrac{b x}{a} = C I$. Also, $A C : A E :: D C : D I$; that is, $a : d :: x : \dfrac{d x}{a} = D I = O E$; and $O E + E p = O p = \dfrac{d x}{a} + c$, and $C E - C I = E I = b - \dfrac{b x}{a} = D O$, since $D O$ is parallel to $C E$; and, by 47th prop. 1st Book of Euclid, $D O^2 + p O^2 = p D^2 = p P^2$; that is, $\left(b - \dfrac{b x}{a}\right)^2 + \left(\dfrac{d x}{a} + c\right)^2 = (c + x)^2$. This, involved and properly reduced, gives $x = \dfrac{a b^2}{2 b^2 + 2 a c - 2 c d} = \dfrac{\frac{1}{2} a (a^2 - d^2)}{a^2 - d^2 + c (a - d)} = \dfrac{\frac{1}{2} a (a + d)}{a + d + c} = D C$, and the whole length of the radius rod $= c + \dfrac{\frac{1}{2} a (a + d)}{a + d + c}$ which is a general expression for every case of this kind.

This expressed in words,—Subtract the square of half the length of the stroke from the square of the length of the beam from the centre of motion, and extract the square root of the remainder; to this root add the above length of the beam, and multiply this sum by half of the above length of the beam, for a dividend.

Then add the above root, the length of the beam, and the distance between the vertical line and the end of the radius rod together, for a divisor.

And if the above dividend be divided by this divisor, the quotient will give the length of the parallel bar; and this, added to the distance between the vertical line and the end of the radius rod, will give the length of the radius rod.

Ex.—Given the length of the beam A C = 12 feet, the length of the stroke C H = 6 feet, and the distance between the vertical line and the end of the radius rod E p = 4 feet, to find the length of the radius rod.

Now, $12^2 = 144$, and $3^2 = 9$; and $144 - 9 = 135$, the square root of which is 11·62 nearly. Then, $11{\cdot}62 + 12 = 23{\cdot}62$; and $23{\cdot}62 \times 6 = 141{\cdot}72$, which is the dividend mentioned in the rule.

Next, $11{\cdot}62 + 12 + 4 = 27{\cdot}62$, the divisor. Therefore, 141·72 divided by 27·62 gives 5·13, the length of the parallel bar; consequently, $5{\cdot}13 + 4 = 9{\cdot}13$ feet, length of radius rod.

This method of finding the length of the radius rod is very useful in rectifying old engines; for the engineer is sometimes compelled to have one end of the radius rod at a given distance from the vertical line which the piston rod is to move in; and we are not aware that it has ever been treated of before by any other author on this subject.

From the above expression, (viz.)

$x = \dfrac{a\,b^2}{2\,b^2 + 2\,a\,c - 2\,c\,\sqrt{(a^2 - b^2)}}$ we may form a general rule, when the length of the stroke, the length of the beam, and the length of the parallel bar are given.

This expression, cleared of fractions, gives $2 b^2 x + 2 a c x - 2 c x \sqrt{(a^2 - b^2)} = a^2 b^2$, or $2 a c x - 2 c x \sqrt{(a^2 - b^2)} = a b^2 - 2 b^2 x = (a - 2 x) b^2$. Hence, $c = \frac{(a - 2 x) b^2}{2 x [a - \sqrt{(a^2 - b^2)}]}$, and $x + \frac{(a - 2 x) b^2}{2 x [a - \sqrt{(a^2 - b^2)}]} = r$, is a general expression for the length of the radius rod. This expression is the same as that given by Mr. Tredgold, in that excellent work which he published on the Steam Engine; but it is obtained here by a process much more simple and elementary.

If, in the above expression, we put $a = 2 x$, we have $x = r$, or the radius rods and parallel rods are of the same length, whatever may be the length of stroke; and it may be remarked, that this is the only case in which the length of the radius rod does not vary with the length of the stroke.

This rule may be expressed in words, for the use of those who do not understand the symbols. From the length of the beam, taken from the centre of motion, subtract twice the length of the parallel bar, and multiply this remainder by the square of half the length of the stroke, for a dividend. Then, from the square of the above length of the beam, subtract the square of half the length of the stroke, and extract the square root of the remainder; subtract this root from the above length of the beam, and multiply the remainder by twice the length of the parallel bar, for a divisor. And if the above dividend by divided by this divisor, and the quotient added to the length of the parallel bar, it will give the length of the radius rod.

Ex.—Given the length of the beam A C from the centre of motion = 12 feet, the length of the stroke C′ H = 6 feet, and the length of the parallel bar C D = 5 feet; to find the length of the radius rod.

Now, $12 - 10 = 2$, and $3^2 \times 2 = 18$, which is the dividend mentioned in the rule.

Then, $\sqrt{(12^2 - 3^2)} = \sqrt{(144 - 9)} = 11{\cdot}62$ nearly, and $12 - 11{\cdot}62 = {\cdot}38$; therefore ${\cdot}38 \times 10 = 3{\cdot}8$, the divisor mentioned in the rule.

$\therefore \frac{18}{3{\cdot}8} + 5 = 9{\cdot}74$, the length of the radius rod required.

In forming these rules, we have assumed the radius rod parallel to the horizon when it has described half the length of its stroke, which is not strictly true, as has been proved; but it is so very near the truth, that no sensible error can possibly arise from that assumption.

Practical Observations.

Since we have given rules for finding the length of radius rods, it now becomes necessary to shew how to put them on.

Plumb the piston rod when the piston is at the top extremity of its stroke; then, one end of the radius rod being moveable about D (Fig. 7 or 8), with the other end p describe the arc $s\,t$. Now, bring the piston down to the lowest extremity of its stroke, and again plumb the piston rod, and in the same manner as before describe the arc $m\,n$; and the point P, where these arcs intersect, is the centre upon which the end P of the radius rod is to move.

It is a practice among engineers to set the cylinder half the vibration of the beam in towards the centre of the beam; and we will here shew how to find the vibration or versed sine of the arc described.

Rule.—From the square of the length of the beam, taken from the centre of motion, subtract the square of half the length of the stroke; and the square root of the remainder, subtracted from the above length of the beam, will give the vibration required.

Ex.—Given the length of the beam from the centre of motion 5 feet, and half the length of the stroke 3 feet, to find the vibration.

Now, $5^2 = 25$, and $3^2 = 9$; then, $\sqrt{(25 - 9)} = \sqrt{16} = 4$.

$5 - 4 = 1$. That is, the vibration will be one inch.

Therefore, in this case, the horizontal distance between the centre of motion and the centre of the cylinder must be 4 feet 11½ inches.

When an engine works with a vibrating pillar, the vibration is in an opposite direction, and the centre of the vibrating pillar axle must be set half the vibration in towards the cylinder.

The air pump bucket rod must be hung on at the point F (Fig. 4); for the point F describes the same kind of a line as the piston rod describes.

In Fig. 9, a parallel motion for a steam-boat engine is represented in three different positions, viz. at the two extremities and at half stroke. H *f* represents the side rod when at the lowest extremity, C′ C represents it when at half stroke, and G C represents it at the top extremity. Also, F *e*, B D, and E *d*, are the corresponding positions of the back link.

Along F *e* take any convenient distance F *k*, and along B D take B *h* = F *k*; also, along E *d* take E *g* = F *k*; and through the three points *g*, *h*, and *k*, describe a circle, the radius of which is the length of the radius rod.

Or, which is the same, from the top of the link in each position take *d g*, D *h*, and *e k*, all equal to each other; and through the three points *g*, *h*, and *k*, describe a circle, and the radius is the required length of the radius rod.

It may be remarked, that the length of the radius rod may be found from this method of construction, whether the back link is parallel to the side rod or not.

To find the Length of the Connecting Rod.

Set the beam at half stroke, that is, parallel to the horizon; and the distance between the centre of the pin on which the connecting rod is to move and the centre of the shaft, is the length of the connecting rod.

Some practical men set both the beam and crank parallel to the horizon, and take the distance between the cen-

tre of the pin on which the connecting rod is to move, and the centre of the crank pin, for the length of the connecting rod.

This method is given in Nicholson's Operative Mechanic and Mechanist, page 170; but it is incorrect, and when the length is so taken, the connecting rod will be found too long.

To construct an Eccentric Wheel.

From the centre of the shaft O (Fig. 9) take O P equal to half the length of the stroke which you intend the wheel to work; and from P as a centre, with any radius greater than P D, describe a circle, and this circle will represent the required wheel. For every circle, drawn from the centre P, will work the same length of stroke, whatever may be its radius; as, whatever you increase the distance of the circumference of the circle from the centre of motion on the one side, you will have a corresponding increase on the opposite side equal to it.

Thus, suppose an eccentric wheel to work a stroke of 18 inches is required, the diameter of the shaft being 6 inches; and if 2 inches be the thickness of metal necessary for keying it on to the shaft, then set off, from O to P, 9 inches; and $9 + 5 = 14$ inches, the radius of the wheel required.

Formulæ.

Let S represent the space the end A is moved through by the eccentric wheel, and s the space the slide moves.

Then, $A\,B \times s = B\,C \times S$; and this equation, solved for A B, B C, S, and s, gives the following:

$$A\,B = \frac{B\,C \times S}{s} \quad (1.) \qquad S = \frac{A\,B \times s}{B\,C} \quad (3.)$$

$$B\,C = \frac{A\,B \times s}{S} \quad (2.) \qquad s = \frac{B\,C \times S}{A\,B} \quad (4.)$$

Ex. 1.—Given the length of the stroke of the slide = 8 inches, the length of the arm B C = 4 inches, and the distance of the centre of the eccentric wheel from the centre of the shaft = 10 inches; required the length of the arm A B.

By Formula 1, $A\,B = \frac{B\,C \times S}{s} = \frac{4 \times 20}{8} = 10$ inches.

Ex. 2.—Given the length of the stroke of the slide = 4 inches, the length of the arm A B = 10 inches, and the eccentricity 6 inches, to find the length of the arm B C.

By Formula 2, $B\,C = \frac{A\,B \times s}{S} = \frac{10 \times 4}{12} = 3\frac{1}{3}$ inches.

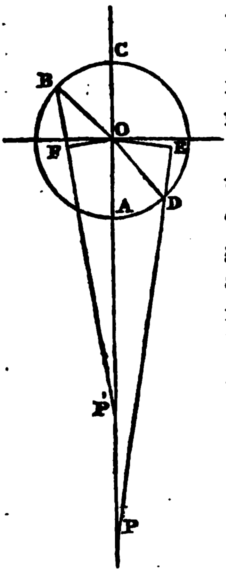
On the Crank.

The crank is used for converting a rectilinear into a rotary motion.

In the crank, as applied in the steam engine, the effect which is produced, is to the effect, were the force to act perpendicularly on the crank all the way round, as twice the diameter of a circle is to the circumference; in consequence of which, many practical men have considered that there is a corresponding loss of power by using a crank; without ever considering that the piston, or moving power, only moves through twice the diameter of the crank's orbit while the crank moves through its whole circumference. For here the same principle holds good as in all other mechanical contrivances, viz. the power multiplied by the space which it passes over, is equal to the weight or resistance multiplied by the space which it passes over.

On the effective Leverage of a Crank.

Let the circle A B C D represent the orbit of the crank, O D any position of the crank below the horizontal line, O A any position above it. Then, to find the effective leverage of the crank when in the position O D, produce the line P D, which represents the connecting rod; and from



the centre O draw O E perpendicular to it; then O E will represent the effective leverage, which may be determined by trigonometry.

For, in the triangle O D P, we have the length of the crank O D, the length of the connecting rod D P, and the angle D O P which the crank makes with a vertical line passing through O, to find the angle O D P. And by Part I. Art. 40, Cor. 3, the effect of a force acting obliquely against a lever is as the sine of the angle of direction which the force makes with the lever; and if the length of the crank be multiplied by the sine of this angle, the product is the effective leverage.

Or it may be done by the following rule:—

Rule.—Multiply the sine of the angle which the crank makes with the vertical line by the length of the crank, and divide this product by the length of the connecting rod, and the quotient will give the sine of the angle which the connecting rod makes with the vertical line. This angle being found, we may find the angle E D O, which is the supplement of the angle which the connecting rod makes with the crank.

$$\text{For, } \angle \text{E D O} = \angle \text{D P O} + \angle \text{D O P}.$$

And if the sine of the angle E D O be multiplied by the length of the crank in inches, the product will give the effective leverage in inches.

Ex.—Given the length of the connecting rod 5 feet, the length of the crank 1 foot, and the angle which it makes with the vertical line 60 degrees, to find the effective leverage.

The natural sine of 60°, the angle which the crank makes with the vertical line, = ·86603.

And, by the rule, $\frac{\cdot 86603 \times 1}{5} = \cdot 173206 =$ sine of the angle which the connecting rod makes with the vertical line, which is, by a table of sines, 9° 38′.

$\angle$ E D O $= \angle$ D P O $+ \angle$ D O P $=$ 9° 58′ + 60° = 69° 58′, the angle which the connecting rod makes with the crank, the natural sine of which is ·93949, the effective leverage; or $\cdot 93949 \times 12 = 11\cdot 27388$ inches.

When the crank is in any position O B above the horizontal line, draw O F perpendicular to B P′, and it will represent the effective leverage of the crank; and to find the angle which the connecting rod makes with the crank, we have the angle B O C which the crank makes with the vertical line A C, the length of the connecting rod, and the length of the crank given. But, in this case, we have $\angle$ B O C $= \angle$ O B P′ $+ \angle$ O P′ B.

$$\therefore \angle \text{O B P}' = \angle \text{B O C} - \angle \text{O P}'\text{ B}.$$

And if the sine of the angle O B P′ be multiplied by the length of the crank in inches, the product is the effective leverage in inches.

Ex.—If the connecting rod be 6 times the length of the crank, and the angle which the crank makes with the vertical line $51\frac{1}{2}$ degrees,

The sine of $51\frac{1}{2}° = \cdot 78261$; and $\cdot 78261 \div 6 = \cdot 13043$ = natural sine of the angle which the connecting rod makes with the vertical line. The corresponding arc is $7\frac{1}{2}$ degrees.

$\angle$ O B P′ $= \angle$ B O C $- \angle$ O P′ B $= 51\frac{1}{2}° - 7\frac{1}{2}° =$ 44°, the angle which the connecting rod makes with the crank, the natural sine of which is ·69466 = the effective leverage; and $\cdot 69466 \times 12 = 8\cdot 33592$ inches.

On the Governor.

In Art. 138, Part I. it was shewn that the periodic time varies as the square root of the altitude, or the altitude varies as the square of the periodic time, let the radius of

the base be what it may. And since the number of revolutions is inversely as the time, the altitude will be inversely as the square of the number of revolutions; and it is known that the distance between the point of suspension and the plane in which the centres of the balls move, is equal to the length of a pendulum which would make two vibrations, viz. one backward and one forward, in the same time that the balls perform one revolution.

Now, the length of a seconds pendulum, or a pendulum which makes 60 vibrations in a minute, is $39\frac{1}{8}$ inches; therefore, if we consider the altitude of a conical pendulum to be $39\frac{1}{8}$ inches, the corresponding number of revolutions in a minute is 30, this being known. Then, as the square of the number of revolutions is to the square of 30, so is $39\frac{1}{8}$ to the altitude required.

Ex.—What is the distance between the point of suspension of a conical pendulum and the plane of revolution, when it makes 25 revolutions per minute?

$25^2 : 30^2 :: 39{\cdot}125 : \frac{35212{\cdot}5}{625} = 56{\cdot}34$ inches, the altitude required.

From this we have the following rule:—Divide 35212·5 by the square of the number of revolutions per minute, and the quotient is the altitude of the conical pendulum.

Ex.—If the length of the arms of a governor or conical pendulum be 30 inches from the point of suspension to the centre of the ball, and the pendulum revolves 40 times in a minute, what will be the diameter of the circle described by the centre of the ball?

$\frac{35212{\cdot}5}{40^2} = 22$ inches, the altitude or distance between the point of suspension and the plane of revolution.

And, to find the radius of the circle described:—From the square of the length of the arm subtract the square of the altitude, and the square root of the remainder will be the radius required. Thus, $\sqrt{(30^2 - 22^2)} = 20{\cdot}4$ inches nearly, the radius of the circle described by the ball.

On Fly-wheels.

The effect of a fly-wheel to remove irregularity in the motion of an engine is proportional to its velocity, if the weight and diameter of the fly be given.

Mr. Buchanan gives the following rule, from Mr. Murray, for finding the weights of fly-wheels:—

Multiply the number of horses' power of the engine by 2000, and divide this product by the square of the velocity of the circumference of the wheel per second; the quotient will give the weight in *cwts.*

Ex.—What must be the weight of a fly-wheel for an engine 20 horse power, 20 feet diameter, and making 20 revolutions in a minute?

20 feet $\times$ 3·1416 $=$ 62·832 feet circumference; and 62·832 $\times$ 20 $=$ 1256·64 feet $=$ the velocity per minute. Therefore, 1256·64 $\div$ 60 $=$ 20·944 feet, velocity per second; and $\frac{20 \times 2000}{(20{\cdot}944)^2} = \frac{40000}{438{\cdot}65} = 91{\cdot}2$ *cwts.* nearly, the weight of the wheel.

Mr. Tredgold's Rule.

Multiply 40 times the pressure on the piston in pounds by the radius of the crank in feet, and divide this product by the cube of the radius of the fly-wheel in feet, and by the number of its revolutions per minute; the result is the area of the rim of the fly in inches.

Ex.—Suppose the pressure on the piston to be 6200 *lbs.* the radius of the crank 3 feet, and the number of revolutions per minute 20; required the area of the section of the rim of the fly of 10 feet radius.

$\frac{6200 \times 40 \times 3}{10^3 \times 20} = \frac{744000}{20000} = 37{\cdot}2$ inches, the area of the section of the rim.

On the Expansive Engine.

Mr. Watt found that by cutting off the steam before the engine had performed its whole length of stroke, a greater quantity of work could be done with the same quantity of steam. His theory of the expansive engine is as follows:—

Let A B C D (Fig. 11) represent a section of the cylinder, and E F the surface of its piston. Let us suppose that the steam is admitted while E F was in contact with A B, and that, as soon as it had pressed it down to the situation E F, the steam cock is shut.

The steam will continue to press it down; and, as the steam expands, its pressure diminishes. We may express its pressure (exerted all the while the piston moves from the situation A B to the situation E F) by the line E F. If we suppose the elasticity of the steam proportional to its density, as is nearly the case with air, we may express the pressure on the piston, such as K L or D C, by K *l* and D *c*, the ordinates of a rectangular hyperbola F *l c*, of which A E and A B are the asymptotes, and A the centre. The accumulated pressure, during the motion of the piston from E F to D C, will be expressed by the area E F *c* D E; and the pressure, during the whole motion, by the area A B F *c* D A.

Now, it is well known that the area E F *c* D E is equal to A B F E × by the hyp. log. $\frac{AD}{AE} = L.\frac{AD}{AE}$, and the whole area A B F *c* D A is $= ABFE\left(1 + L.\frac{AD}{AE}\right)$

Thus, let the diameter of the piston be 24 inches, and the pressure of the atmosphere on a square inch be 14 *lbs.*; the pressure on the piston is 6333 *lbs.*; let the whole stroke be 6 feet; and let the steam be stopped when the piston has descended 18 inches, or 1·5 foot. The hyperbolic lo-

garithm of $\frac{6}{1\cdot5}$ is 1·3862948; therefore the accumulated pressure A B F *c* D A is $6333 \times 2\cdot3862948 = 15114$ *lbs.* Take the common logarithm of $\frac{\text{A D}}{\text{A E}}$, and multiply it by 2·3026; the product is the hyp. log. $\frac{\text{A D}}{\text{A E}}$

The accumulated pressure while the piston moves from A B to E F is 6333×1, or simply 6333 *lbs.*; therefore the steam, while it expands into the whole cylinder, adds a pressure of 8781 *lbs.*

Suppose the steam had got free admission during the whole descent of the piston, the accumulated pressure would have been $6333 \times 4 = 25332$ *lbs.* Here Mr. Watt observed a remarkable result. The steam expended in this case would have been four times greater than when it was stopped at one-fourth, and yet the accumulated pressure is not twice as great, being nearly five-thirds. One-fourth of the steam performs nearly three-fifths of the work, and an equal quantity performs more than twice as much work, when thus admitted during one-fourth of the motion.

Note.—All these calculations, however, proceed upon the supposition that steam contracts and expands by variations of pressure in the same ratios that air would do.

Summary of Proportions of a double Engine, working at full Pressure.

The length of a cylinder should be twice its diameter; for a cylinder having this proportion exposes less surface to condensation than any other, enclosing the same quantity of steam.

The area of the steam passages should be about one-fifth of the diameter of the cylinder; or their area should be equal to the area of the cylinder, multiplied by the velo-

city of the piston in feet per minute, and divided by 4800.—*Tredgold.*

The diameter of the air pump should be about two-thirds of the diameter of the cylinder, and half the length of stroke; and the larger the passages through the air bucket and the discharging flap are, the better.

The quantity of water for injection should be about 23½ times that required for steam, or about 26 cubic inches to each cubic foot of the contents of the stroke of the piston. Mr. Watt considered a wine pint, or 28⅞ cubic inches, quite sufficient.

There should be 62 times as much water in the boiler as is introduced at one feed.—*Tredgold.*

Mr. Tredgold gives the following method of determining the number of horses' power:—

If the force of the steam in the boiler be denoted by		1·000
Then, besides the loss from uncondensed steam, there is a loss,		
First, by the force producing the motion of the steam into the cylinder, ...	·007	
Second, by cooling in the cylinder and pipes,	·016	
Third, by the friction of the piston and loss,	·125	
Fourth, by the force necessary to expel the steam through the passages, ...	·007	
Fifth, by the force required to open and close the valves, raise the injection water, and the friction of the axis, ...	·063	
Sixth, by the steam being cut off before the end of the stroke, ...	·100	
Seventh, by the power necessary to work the air pump, ...	·080	
	nearly	·400
		·600

From the above it appears that there is a loss of about $\frac{4}{10}$ of the force of steam in the boiler, and therefore only $\frac{6}{10}$ of that force is effective.

Rule.—Multiply the number of square inches in the area of the piston by the mean effective pressure on each square inch, and that product by the velocity in feet per minute; the result will be the effective power in pounds raised one foot high per minute. To find the horses' power, divide this result by 33000.

Some engineers consider that a horse can draw 200 *lbs.* at the rate of 220 feet per minute, over a pulley; and if this be taken as a standard, a horse will draw 44000 *lbs.* one foot high in a minute.

The force of the steam in the boiler is generally 35 inches of mercury, the temperature of the uncondensed steam 120°, its force 3·7 inches. Hence, $35 \times \cdot 6 = 21\cdot 0$; and $21\cdot 0 - 3\cdot 7 = 17\cdot 3$ inches, which is about $8\frac{1}{2}$ *lbs.* per square inch for the mean effective pressure.

Ex.—Given the diameter of the cylinder of a double acting steam engine 24 inches, the length of the stroke 5 feet, and the number of strokes per minute 22; find the horses' power, if the mean effective pressure on each square inch is 9 *lbs.*

The velocity of the piston is $2 \times 5 \times 22 = 220$ feet per minute.

Then, $24^2 \times \cdot 7854 \times 9 \times 220 = 895733$ *lbs.* nearly, raised one foot high per minute.

$$\text{And } \frac{895733}{33000} = 27 \text{ horses' power.}$$

$$\text{Or, } \frac{895733}{44000} = 20\cdot 3.$$

Note.—We have increased the loss of power in working the air pump, as we are fully persuaded that Mr. Tredgold's rule makes the diameter of the air pump too small.

Mr. Tredgold gives the following method of calculating the power of a high pressure engine:—

First, by the force producing motion of the steam into the cylinder, ...	·0069
Second, by the cooling in the cylinder and pipes,	·0160
Third, by the friction of the piston and waste, ...	·2000
Fourth, by the force required to expel the steam into the atmosphere, ...	·0069
Fifth, by the force expended in opening valves, and friction of the parts of the engine, ...	·0622
Sixth, by the steam being cut off before the termination of the stroke, ...	·1000
	·3920

We may consider this 0·4, and then the effective pressure is 0·6 of the force of the steam in the boiler, diminished by the pressure of the atmosphere; whence we have the following rule for high pressure engines, working at full pressure:—

Rule.—Multiply six-tenths of the excess of the force of the steam in the boiler, less four-tenths of the pressure of the atmosphere in pounds on a square inch, by the area of the cylinder in square inches, and the velocity of the piston in feet per minute; the product is the power of the engine in pounds raised one foot high per minute.

To find its equivalent in horses' power, divide by 33000.

Ex.—Given the diameter of the cylinder 12 inches, the length of the stroke 3 feet, the number of strokes per minute 30, and the force of the steam in the boiler 30 *lbs.* per square inch above the pressure of the atmosphere; required the number of horses' power.

The velocity of the piston $2 \times 3 \times 30 = 180$ feet per minute.

And $(30 \times 0{\cdot}6) - (15 \times 0{\cdot}4) = 12$; then, $12 \times 12^2 \times {\cdot}7854 \times 180 = 244290{\cdot}8$ *lbs.* raised one foot high per minute.

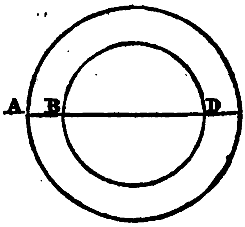
$$\frac{244290{\cdot}8}{33000} = 7\tfrac{1}{2} \text{ horses' power nearly.}$$

On Winding Engines.

In winding engines drawing coals out of a pit, where we intend them to go a given number of strokes in drawing a corf, we must ascertain the diameter of the roll at first lift. In this case, we suppose the engine to have flat ropes, such as are generally used, and which lay upon each other.

To find the diameter of a rope roll at the first lift, it is necessary to know the depth of the pit, the thickness of the rope, and the number of strokes which you intend the engine to make in drawing up a corf or corves.

Then, the thickness of the rope being known, and the number of strokes, we can determine the thickness of rope upon the roll, let the diameter of the roll be what it may. Thus, suppose the thickness of the rope to be one inch, and the number of strokes 10; then the radius of the roll is increased 10 inches, or the diameter is increased 20 inches, whatever that diameter may be.



Let x = B D, the diameter of the roll at first lift, t = the thickness of the rope, b = A B, the number of inches the radius of the roll is increased, and $p = \cdot 7854$.

Then, $p\,x^2$ = area of the small circle, and $p\,(x + 2\,b)^2 = p\,x^2 + 4\,p\,b\,x + 4\,p\,b^2$ = the area of the large circle; their difference is $4\,p\,b\,x + 4\,p\,b^2$ = the area of the annulus.

But the area of the annulus is equal to the area of the edge of the rope; that is, the depth of the pit multiplied by the thickness of the rope. Let the depth of the pit in inches = d; then $d\,t$ = area of the edge of the rope.

$$\therefore\ 4\,p\,b\,x + 4\,p\,b^2 = d\,t.$$

$$\text{Or } x = \frac{d\,t - 4\,p\,b^2}{4\,p\,b} = \frac{d\,t}{4\,p\,b} - b.$$

Hence we have the following rule:—

Multiply the depth of the pit, in inches, by the thickness of the rope, also in inches, for a dividend.

Then multiply 3·1416 times the thickness of the rope, in inches, by the number of strokes, for a divisor.

Divide the above dividend by this divisor, and from the quotient subtract the product which is found by multiplying the thickness of the rope by the number of strokes, and the remainder will give the diameter of the roll in inches.

Ex.—If an engine makes 20 strokes in drawing a corf up a pit, the depth of which is 100 fathoms, and the thickness of the rope one inch, what is the diameter of the roll at the first lift?

100 fathoms $=$ 7200 inches, and $7200 \times 1 = 7200$, which is the dividend mentioned in the rule.

And $3·1416 \times 1 \times 20 = 62·832$, the divisor mentioned in the rule.

$7200 \div 62·832 = 114·6$ nearly, and $114·6 - 20 = 94·6$ inches $= 7$ feet $10\frac{6}{10}$ inches.

It may be remarked here, that if an engine be drawing coals out of a pit with round ropes, and you wish to take the round ropes off and to put flat ones on, this rule will determine what diameter your roll must be at first lift, so that the engine may go the same number of strokes as before, when the round ropes were on.

Ex.—If an engine goes 10 strokes in drawing a corf up a pit, the depth of which is 60 fathoms, with round ropes, where the round ropes do not lay upon each other, what must be the diameter of a flat rope roll, so that the engine may go the same number of strokes as before, the thickness of the rope being half an inch?

60 fathoms $=$ 4320 inches.

$4320 \times \frac{1}{2} = 2160$, the dividend mentioned in the rule.

$3·1416 \times \frac{1}{2} \times 10 = 15·708$, the divisor which is mentioned in the rule.

And the product of the thickness of the rope and number of strokes is $\frac{1}{2} \times 10 = 5$.

Hence, $137\cdot5 - 5 = 132\cdot5$ inches $= 11$ feet $0\frac{1}{2}$ inches.

When an engine draws coals out of the pit with flat ropes, the corves will not pass each other at mid-shaft, that is, half way between the top and bottom of the pit; for the corf which goes from the top of the pit will pass through a greater space in the same time, than the corf which comes from the bottom, owing to the circumference of the roll being always greater until the engine has performed half her number of strokes. Therefore, meetings will always be below mid-shaft. At meetings each roll is equal; and, after this, the roll on which the ascending corf hangs continues to increase until the corf arrives at the top of the pit.

The diameter of the roll evidently increases in arithmetical progression; and if the diameter be multiplied by 3·1416, the product gives the circumference of the roll, or the length of the first turn.

Now, suppose the engine makes any number of strokes, take 16 for example; and if the diameter of the roll be 9 feet, and the thickness of the rope one inch, then the roll when full will be 11 feet 8 inches in diameter. But to find the length of the last turn, we must take an inch off each side; then the diameter for finding the last turn will be 11 feet 6 inches, and this multiplied by 3·1416 gives the length of the last turn.

Then, add the length of the first turn and the length of the last turn together, and multiply this sum by half the number of turns, and this product will give the sum of all the turns, or the depth of the pit.

Ex.—If an engine makes 16 strokes in drawing a corf of coals, the diameter of the roll being 9 feet at first lift, and thickness of the rope one inch; what will be the depth of the pit, and how far below mid-shaft will the corves meet?

$3{\cdot}1416 \times 9 = 28{\cdot}2744 =$ circumference of the roll, or the length of the first turn in feet.

To find the last turn, add 15 inches to the radius of the roll, or 30 inches to the diameter; then, $9 + 2\frac{1}{2} = 11\frac{1}{2}$, and $3{\cdot}1416 \times 11\frac{1}{2} = 36{\cdot}1284 =$ length of the last turn in feet.

$36{\cdot}1284 =$ length of the last turn in feet.
$28{\cdot}2744 =$ length of the first turn in feet.

$64{\cdot}4028 =$ sum of the first and last turns.
$8 =$ half the number of turns.

$6)515{\cdot}2224 =$ depth of the pit in feet, or the whole length of the rope on the roll.
$85{\cdot}8704 =$ depth of the pit in fathoms.

To find meetings:—When the engine goes 8 strokes, the corves will be at meetings; therefore, add 7 times the thickness of the rope to the radius, or 14 times its thickness to the diameter of the roll; that is, $9 + 1\frac{1}{8} = 10\frac{1}{8}$ feet $=$ diameter of the last turn to meetings; and $10\frac{1}{8} \times 3{\cdot}1416 = 31{\cdot}9396 =$ circumference of the last turn to meetings.

$31{\cdot}9396 =$ length of the last turn to meetings.
$28{\cdot}2744 =$ circumference of roll, or length of first turn.

$60{\cdot}2140$
$4 =$ half the number of turns to meetings.

$6)240{\cdot}8560 =$ distance in feet of meetings from the bottom of the pit.
$40{\cdot}1426 =$ fathoms.

Now, to find the distance from the top of the pit to meetings, $85{\cdot}8704 - 40{\cdot}1426 = 45{\cdot}7277$ fathoms.

But it may be found in the same manner as above, thus: The corves will be at meetings when 8 turns have gone off the full roll; therefore we must find the length of the

eighth turn from the top; but the eighth turn from the top is the ninth turn from the roll, consequently we must add 16 times the thickness of the rope to the diameter of the roll.

16 inches $= 1\frac{1}{3}$ feet.

$9 + 1\frac{1}{3} = 10\frac{1}{3}$ = the diameter of the last turn that goes off the roll to meetings.

$10\frac{1}{3} \times 3{\cdot}1416 = 32{\cdot}4632$ = length of the last turn that goes off to meetings.

$36{\cdot}1284$ = length of the first turn that goes off.
$32{\cdot}4632$ = length of last turn to meetings.

$68{\cdot}5916$
4 = half the number of turns to meetings.

$6)274{\cdot}36640$ = distance from the top of the pit to meetings in feet.

$45{\cdot}72773$ = that distance in fathoms, the same as above.

$$\begin{array}{r} 45{\cdot}72773 \\ 40{\cdot}14266 \\ \hline \end{array}$$

Meetings will be $5{\cdot}58306$ fathoms below mid-shaft.

PART III.

HYDROSTATICS.

HYDROSTATICS treats of the pressure, weight, and equilibrium of non-elastic fluids.

If a fluid be at rest in a vessel, the base of which is parallel to the horizon, equal parts of the base are equally pressed by the fluid.

For, upon every part of the base there is an equal column of the fluid supported; and as all the columns are of equal weight, they must press the base equally, or equal parts of the base will sustain an equal pressure.

All the parts of the fluid press equally at the same depth.

The pressure of a fluid at any depth is as the depth of the fluid.

For the pressure is as the weight, and the weight is as the height of a column of the fluid.

If a fluid is pressed by its own weight, or by any other force, at any point it presses equally in every direction whatever.

This arises from the nature of fluidity, which is, to yield to any force in any direction. If it cannot give way to any force which is applied, it will press against other parts of the fluid in the direction of that force; and the pressure in all directions is the same; for if any one was less, the fluid would move that way until the pressure was the same every way.

Therefore, in a vessel containing a fluid, the pressure is the same against the bottom as against the sides, or even upwards, at the same depth.

The pressure of a fluid upon the base of the containing vessel, is as the base and perpendicular altitude, whatever may be the figure of the containing vessel.

The pressure of a fluid against any upright surface, as the gate of a sluice, is equal to the area of that surface multiplied by half its depth.

Ex.—If the gate of a sluice be 9 feet broad and 6 feet deep, what is the pressure of water against it?

$9 \times 6 \times 3 = 162 =$ the area multiplied by half the depth.

$$\frac{162 \times 1000}{16} = 10125 \textit{ lbs.} \text{ or } 4{\cdot}5 \text{ tons.}$$

The pressure against the internal surface of a cubical vessel is three times the weight of the fluid contained in it; for the pressure against one side of the vessel is equal to half the pressure on the bottom.

Ex.—What pressure does the internal surface of a cubical vessel sustain, each side of the cube being 4 feet?

$$4^3 \times 62\tfrac{1}{2}^{*} = 4000, \text{ and } 4000 \times 3 = 12000.$$

* $62\frac{1}{2}$ *lbs.* is the weight of a cubic foot of water.

On the Hydrostatic Paradox.

The hydrostatic paradox may be explained upon the same principles as the mechanical powers; and an explanation conducted in this manner strips it of its paradoxical appearance.

The hydrostatic paradox is expressed thus:—A quantity of fluid, however small, may be made to balance a quantity, however large. Thus, in Fig. 1 to *Hydrostatics*, let A B be a large vessel, and C D a small one which is connected with it. Then, if water be poured into either of them, it will stand at the same height in both; consequently there is an equilibrium between them.

Now, it must be observed that there is an equilibrium between them in the same manner as in any of the mechanical powers. Take the lever, for instance: suppose 1 *lb.* to balance 100 *lbs.*; this is exactly the same as 1 *lb.* balancing 100 *lbs.* in the hydrostatic paradox. In the former case, the length of the arms of the lever must be in proportion to each other as 1 to 100; and, in the latter, the area of the vessels must be to each other as 1 to 100. But, properly speaking, one pound does not, nor cannot balance one hundred, in any case whatever; for one pound can only balance one pound, use what means you will.

Archimedes only required a fixed point to be able to sustain the whole earth; but, as Carnot very justly observed, if he had found it, it would not, in fact, have been Archimedes, but the fixed point or fulcrum, which would have sustained the earth.

In the hydrostatic paradox also, one pound, instead of balancing one hundred, only balances one pound. All the rest of the weight is supported by the vessel, in the same manner as the weight is supported on the fulcrum of the lever.

And, to shew that the same takes place as in the lever, fix a piston which is water-tight into either of them, suppose

the greater for instance; and if the area of the greater be to the area of the lesser as 100 to 1, then, by pressing the piston down 1 inch in the large vessel, it will raise the water 100 inches in the smaller one. Now, this is a property of the lever which is well known to every one, viz. that if the arms of the lever be as 1 to 100, then the long end of the lever will move through 100 inches whilst the short one moves through 1 inch.

It is upon the principle of the hydrostatic paradox that Mr. Bramah applied the hydrostatic press.

Suppose the diameter of the cylinder piston to be 20 inches, and that of the pump piston which is used for forcing the water into the cylinder $\frac{1}{4}$ of an inch; then, the area of the cylinder piston is $20^2 \times \cdot 7854 = 314 \cdot 16$ square inches, and the area of the pump piston is $(\frac{1}{4})^2 \times \cdot 7854 = \frac{1}{16} \times \cdot 7854 = \cdot 0491$.

Then, $\dfrac{314 \cdot 16}{\cdot 0491} = 6400$; that is, the area of the cylinder piston is to the area of the pump piston as 6400 to 1. Therefore, if any force be applied to the small piston, it will produce an effect on the large one as 1 to 6400, if there be water or any incompressible fluid whatever between them.

Thus, suppose, by means of a lever, we can press down the small piston with a force of 10 *cwt.*; then the large piston will ascend with the force of 10×6400 *cwt.* $= 3200$ tons; and in the same manner the power may be calculated, whatever proportion the pistons may have to each other.

On Specific Gravity.

The specific gravity of a body is the relation of its weight, compared with the weight of some other body of the same magnitude.

A body immersed in a fluid will sink if its specific gravity be greater than that of the fluid; but if it be less, the

body will rise to the top, and will be only partly immersed.

If the specific gravity of the body and fluid are equal, then the body will remain at rest in any part of the fluid.

If the body be heavier than the fluid, it loses as much of its weight, when immersed, as is equal in weight to a quantity of the fluid of the same bulk.

If the specific gravity of the fluid be greater than that of the body, then the quantity of the fluid displaced by the part immersed is equal in weight to the weight of the whole body. Therefore, the specific gravity of the fluid is to that of the body, as the whole magnitude of the body is to the part immersed.

The specific gravities of equal solids are as their parts immersed in the same fluid.

The specific gravities of fluids are as the weights lost by the same immersed body.

When the Body is heavier than Water,

Rule.—Weigh it both in and out of water, and then say,

As the weight lost in water,
Is to the whole or absolute weight;
So is the specific gravity of water,
To the specific gravity of the body.

Ex.—Required the specific gravity of a piece of tin which weighs 23 *lbs.* but in water only 20 *lbs.*; the specific gravity of water being 1000.

$$23 - 20 = 3 \text{ lbs. weight lost in water.}$$

$$3 : 23 :: 1000 : 7333 = \text{the specific gravity.}$$

When the Body is lighter than Water,

Rule.—Attach to it a piece of another body, heavier than water, so that the mass compounded of the two may sink together. Weigh the denser body and the compound body separately, both out of the water and in it, and find how much each loses in the water by subtracting its weight in water from its weight in air; and subtract the less of these

remainders from the greater. Then use the following proportion:

As the last remainder,
Is to the weight of the light body in air;
So is the specific gravity of water,
To the specific gravity of the body.

Ex.—If a piece of elm weighs 15 *lbs.* in air, attached to which is a piece of copper which weighs 18 *lbs.* in air, and 16 *lbs.* in water, and the compound weighs 6 *lbs.* in water, what is the specific gravity of the elm?

18 + 15 = 33 *lbs.* weight of the compound in air.

33 — 6 = 27 *lbs.* weight which the compound loses in water.

18 — 16 = 2 *lbs.* weight which the copper loses in water.

And 27 — 2 = 25.

25 : 15 :: 1000 : 600 = specific gravity of the elm.

To find the Specific Gravity of a Fluid.

Rule.—Take a piece of some body of known specific gravity, weigh it both in and out of the fluid, and find the loss of weight by taking the difference of these two. Then say,

As the whole or absolute weight,
Is to the loss of weight;
So is the specific gravity of the solid,
To the specific gravity of the fluid.

Ex.—A piece of granite weighs 6 *lbs.* in a fluid, and 9 *lbs.* out of it; what is the specific gravity of that fluid?

9 — 6 = 3 *lbs.* weight lost in fluid.

Then, 9 : 3 :: 3000 : 1000 = the specific gravity of the fluid.

To find the Quantities of two Ingredients in a given Compound.

Rule.—Take the three differences of every pair of the three specific gravities, viz. the specific gravities of the

compound and each ingredient, and multiply each specific gravity by the difference of the other two. Then say,

As the greatest product,
Is to the whole weight of the compound;
So is each of the other two products,
To the weights of the two ingredients.

Ex.—A composition of 112 *lbs.* being made of tin and copper, the specific gravity of which is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and that of copper 9000.

8784 composition, 9000 copper, 7320 tin.

$9000 - 7320 = 1680$, and $1680 \times 8784 = 14757120$.
$9000 - 8784 = 216$, and $216 \times 7320 = 1581120$.
$8784 - 7320 = 1464$, and $1464 \times 9000 = 13176000$.
Then, $14757120 : 112 :: 13176000 : 100 =$ the copper.
$14757120 : 112 :: 1581120 : 12 =$ tin.

To find the Magnitude of a Body from its Weight being given, say,

As the specific gravity of the body,
Is to its weight in avoirdupois ounces;
So is a cubic foot, or 1728 cubic inches,
To the solid contents of the body in feet or inches.

Ex.—Required the contents of a piece of dry oak, the specific gravity of which is 925, and its weight 1000 ounces.

$$925 : 1000 :: 1 : \frac{1000}{925} = 1{\cdot}189 \text{ cubic feet.}$$

To find the Weight of a Body from its Magnitude being given, say,

As one cubic foot, or 1728 inches.
Is to the solid content of the body;
So is the specific gravity of the body,
To its weight in avoirdupois ounces.

Ex.—Given the diameter of a cast iron cylinder 20 inches, the thickness of the metal 1 inch, and the length of the cylinder 5 feet, to find its weight.

$\therefore\ 20^2 \times \cdot 7854 = 314\cdot 16 =$ area of the inside.

Now, to find the outside diameter, we must add 1 inch to each side; then, 22 inches is the outer diameter, and $22^2 \times \cdot 7854 = 380\cdot 1336$.

Then, $380\cdot 1336 - 314\cdot 16 = 65\cdot 9736 =$ area of the metal.

And $65\cdot 9736 \times 60 = 3958\cdot 416 =$ solid inches of metal.

$1728 : 3958\cdot 416 :: 7207 : 16508\cdot 8$ ounces.

$16508\cdot 8 \div 16 = 1031\cdot 8$ *lbs.*

Examples for Practice.

Ex. 1.—How deep will a cube of oak sink in common water; each side of the cube being 1 foot, spec. grav. $=$ 925?

Since the specific gravity of the fluid is to the specific gravity of the body, as the magnitude of the body to the magnitude of the part immersed, we have,

$1000 : 925 :: 1728 : 1598.4 =$ solidity or magnitude of the part immersed.

Hence, we have given the contents $= 1598\cdot 4$, and the area of the end $= 144$, to find the height.

$$\frac{1598\cdot 4}{144} = 11\cdot 1 \text{ inches, the depth required.}$$

Or thus:

Since the weight of the water displaced is equal to the weight of the body, we have the weight of the water displaced given $= 925$ ounces, and the area of the end $=$ 144 square inches, to find the depth.

A cubic inch of water weighs $\cdot 5787$ ounces avoirdupois.

$\therefore\ 925 \div \cdot 5787 = 1598\cdot 4$ cubic inches, the content of the water displaced.

$1598\cdot 4 \div 144 = 11\cdot 1$ inches.

That is, a column of water one foot square, and $11\frac{1}{10}$ inches deep, will weigh 925 ounces.

If it were required to know what weight must be added to it, so as to make the upper surface level with the water:—

The weight of the water displaced is equal to the weight of the body; but when the upper surface is level with the water, there is a cubic foot of water displaced, the weight of which is 1000 ounces. Therefore it will require as much weight to be added to it as to make the weight of the body 1000 ounces.

1000 — 925 = 75 ounces = the weight which must be added to it, to make the upper surface level with the surface of the water.

Ex. 2.—Suppose, by measurement, it be found that a man-of-war, with its ordnance, rigging, and appointments, sinks so deep as to displace 1300 tuns of sea water; what is the whole weight of the ship, supposing a cubic inch of sea water to weigh ·5949 of an ounce avoirdupois?

The weight of the water displaced is equal to the weight of the ship.

216 gallons = 1 tun.

1300 × 216 = 280800 gallons; and if we take 277·2738 cubic inches to the gallon, then 280800 × 277·2738 = 77858483.04 cubic inches, and this multiplied by ·5949 gives 46318011·5367 ounces = 1292·35 tons, the weight of the ship.

Ex. 3.—An irregular fragment of glass, in the scale, weighs 171 grains, and another of magnet 102 grains; but in water the first fetches up no more than 120 grains, and the other 79; what then will their specific gravities turn out to be?

The specific gravities of different bodies in the same fluid are as $\frac{w}{w'}$; w being the weight of the body, and w' the weight lost in the fluid.

$$\frac{171}{51} : \frac{102}{23} :: 3933 : 5202.$$

That is, the specific gravity of glass is to that of water as 3933 : 5202.

Ex. 4.—Supposing the cubic inch of common glass weighs 1·4921 ounces troy, the same of sea-water ·59542, and of brandy ·5368; then a seaman having a gallon of this liquor in a glass bottle, which weighs 3·84 *lbs.* out of water, and, to conceal it from the officers of the customs, throws it overboard. It is proposed to determine, if it will sink, how much force will just buoy it up?

$$3{\cdot}84\ lbs. \times 12 = 46{\cdot}08 \text{ ounces, and } \frac{46{\cdot}08}{1{\cdot}4921} = 30{\cdot}88265 =$$

cubic inches in the bottle.

If we suppose a gallon of brandy to contain 231 solid inches, then $231 + 30{\cdot}8826 = 261{\cdot}8826$ cubic inches in both.

$261{\cdot}8826 \times {\cdot}59542 = 155{\cdot}9311$ ounces nearly, the weight of the salt water occupied by both bottle and brandy.

Then, ${\cdot}5368 \times 231 = 124{\cdot}0008$ ounces, weight of the brandy.

$124{\cdot}0008 + 46{\cdot}08 = 170{\cdot}0808$ ounces = the weight of both.

$170{\cdot}0808 - 155{\cdot}9311 = 14{\cdot}1497$ ounces.

Ex. 5.—A given cone is immersed in water, with its vertex downwards; what part of the axis will be immersed if the specific gravity of the fluid be to that of the cone as 8 to 5?

Now, by the principles of hydrostatics, the magnitude of the whole cone is to the magnitude of the part immersed, as the specific gravity of the fluid is to that of the body, or as 8 to 5. But the whole cone and part immersed are similar; their contents are as 8 to 5. Hence, $\sqrt[3]{8} : \sqrt[3]{5} :: 1 : \frac{1}{2}\sqrt[3]{5} = {\cdot}854988$, which expresses the part of the axis immersed, the whole being 1.

Ex. 6.—Hiero, king of Sicily, ordered his jeweller to make him a crown, containing 63 ounces of gold. The workmen thought that substituting part silver was only a

proper perquisite; which taking air, Archimedes was appointed to examine it; who, on putting it into a vessel of water, found it raised the fluid 8·2245 cubic inches: and having discovered that the inch of gold more critically weighed 10·36 ounces, and that of silver but 5·85 ounces, he found by calculation what part of the king's gold had been changed. And you are desired to repeat the process.

$$10{\cdot}36 : 1 :: 63 : 6{\cdot}081 \text{ cubic inches.}$$
$$5{\cdot}85 : 1 :: 63 : 10{\cdot}77 \text{ cubic inches.}$$

By Alligation:

$$8{\cdot}2245\left\{\begin{matrix} 6{\cdot}081 \\ 10{\cdot}77 \end{matrix}\right\}\begin{matrix} 2{\cdot}5455 \text{ cubic inches gold.} \\ 2{\cdot}1435 \text{ cubic inches silver.} \end{matrix}$$

$$2{\cdot}5455 + 2{\cdot}1435 = 4{\cdot}689.$$

Then, $4{\cdot}689 : 63 :: 2{\cdot}5455 : 34{\cdot}2$ ounces gold.

$4{\cdot}689 : 63 :: 2{\cdot}1435 : 28{\cdot}8$ ounces silver.

Ex. 7.—What must be the thickness of a right-angled cone of copper, the inner diameter of which is 20 inches, so that it may just float with its edge level with the surface of the fluid; the specific gravities of the copper and fluid being as 9 to 1, and the interior and exterior surfaces having a common base?

Let r = inner radius, t = thickness, and $p = 3{\cdot}1416$. Then, since the cone is right-angled, the radius of the base is equal to the altitude; therefore,

$$\frac{p\,(r+t)^2 \times (r+t)}{3} = 9.\ \frac{p\,[(r+t)^3 - r^3]}{3}$$

$$\therefore (r+t)^3 = 9\,(r+t)^3 - 9\,r^3.$$

$$8\,(r+t)^3 = 9\,r^3.$$

$$2\,(r+t) = r\,\sqrt[3]{9}.$$

$$t = \frac{r\,(\sqrt[3]{9} - 2)}{2} = \frac{10\,(\sqrt[3]{9} - 2)}{2} = \cdot 40042 \text{ parts of}$$

an inch.

TABLES OF SPECIFIC GRAVITIES.

SOLIDS.

Platina	20,722
Gold, pure, hammered	19,362
Guinea of George III.	17,629
Tungsten	17,600
Mercury, at 32° Fahr.	13,598
Lead	11,352
Palladium	11,300
Rhodium	11,000
Virgin Silver	10,744
Shilling of George III.	10,534
Bismuth, molten	9,822
Copper, wiredrawn	8,878
Red Copper, molten	8,788
Molybdena	8,611
Arsenic	8,308
Nickel, molten	8,279
Uranium	8,100
Steel from 7,767 to	7,816
Cobalt, molten	7,812
Bar Iron	7,788
Pure Cornish Tin	7,291
Do. hardened	7,299
Cast Iron	7,207
Zinc	6,862
Antimony	6,712
Tellurium	6,115
Chromium	5,900
Spar, heavy	4,430
Jargon of Ceylon	4,416
Oriental Ruby	4,283
Sapphire, Oriental	3,994
Do. Brazilian	3,131
Oriental Topaz	4,019
Oriental Beryl	3,549
Diamond from 3,501 to	3,531
English Flint-Glass	3,329
Tourmalin	3,155
Asbestus	2,996

Marble, green, Campan.	2,742
——— Parian	2,837
——— Norwegian	2,728
——— green, Egyptian	2,668
Emerald	2,775
Pearl	2,752
Chalk, British	2,784
Jasper	2,710
Coral	2,680
Rock Crystal	2,653
English Pebble	2,619
Limpid Feldspar	2,564
Glass, green	2,642
—— white	2,892
—— bottle	2,733
Porcelain, China	2,385
——— Limoges	2,341
Native Sulphur	2,033
Ivory	1,917
Alabaster	1,874
Alum	1,720
Copal, opaque	1,140
Sodium	973
Oak, heart of,	950
Gunpowder, about	937
Ice	930
Potassium	866
Beech	852
Ash	845
Apple-Tree	793
Orange-Wood	705
Pear-Tree	661
Linden-Tree	604
Cypress	598
Cedar	561
Fir	550
Poplar	383
Cork	240

LIQUIDS.

Sulphuric Acid	1,841	Burgundy Wine -	991
Nitrous Acid -	1,550	Olive Oil - -	915
Water from the Dead Sea - -	1,240	Muriatic Ether -	874
Nitric Acid -	1,218	Oil of Turpentine	870
Sea-Water -	1,026	Liquid Bitumen -	848
Milk - -	1,030	Alcohol, absolute	792
Distilled Water	1,000	Sulphuric Ether -	716
Wine of Bourdeaux	994	Air at the Earth's Surface, about -	1¼

*** Since a cubic foot of water, at the temperature 40° Fahrenheit, weighs 1000 *oz.* avoirdupois, or 62½ *lbs.* the numbers in the preceding Tables exhibit very nearly the respective weights of a cubic foot of the several substances tabulated.—*Dr. Gregory's Edition of Dr. Hutton's Course.*

HYDRAULICS.

Hydraulics or Hydrodynamics treats of the motion of fluids, and the forces with which they act upon bodies against which they strike, or which move in them.

The velocity with which a fluid issues from a very small orifice in the bottom or side of a vessel that is kept constantly full, is equal to that which a heavy body would acquire by falling from the level of the surface of the fluid to the level of the orifice.

Therefore, if h = height of the fluid above the orifice, g = the velocity acquired by a falling body in one second, and v = the velocity with which the water issues,

$$v = \sqrt{2 g h}.$$

The quantity of water that issues in one second through a given orifice is equal to a column of water having the area of the orifice for its base, and the velocity with which the fluid issues for its altitude.

That is, if A = the area of the orifice, and Q = the quantity of fluid running out in one second,

$$Q = A \sqrt{2 g h}.$$

$$\text{Or, if Q and } h \text{ be given, then } A = \frac{Q}{\sqrt{2gh}};$$

$$\text{And if Q and A be given, } h = \frac{Q^2}{2gA^2}$$

Experiments do not exactly agree with this theory as to the quantity of water run out; for the vein of water that issues through the small orifice suffers a contraction, by which its section has been found to be diminished in the ratio of nearly 5 to 7. Therefore, instead of $Q = A\sqrt{2gh}$, we must take $Q = \frac{5}{7} A \sqrt{2gh}$. But $\frac{5}{7}$ is nearly $= \frac{1}{\sqrt{2}}$; therefore, we may take $Q = \frac{1}{\sqrt{2}} \cdot A\sqrt{2gh} = \frac{1}{\sqrt{2}} \times A \times \sqrt{2} \times \sqrt{gh} = A\sqrt{gh}$. But $\sqrt{gh}$ is the velocity which a heavy body acquires in falling through $\frac{h}{2}$; consequently, the velocity of the water at the orifice is found equal to that which a heavy body would acquire in falling freely through half the altitude.

The experiments of Bossut show that the actual discharge through a hole made in the side or bottom of a vessel, is to the theoretical as 1 to ·62, or nearly as 8 to 5. Consequently, the theoretical discharge must be diminished in this ratio to have the true discharge. Also, if a pipe from 1 to 2 inches long be inserted in the aperture, the contraction of the vein is prevented, and the actual discharge is to the theoretical as 4 to 5.

The quantities discharged are as the square root of the depth multiplied into the areas of the orifices.

The following is extracted from Mr. Banks's Treatise on Mills:—

When the water is discharged through perpendicular sections, the velocity at the bottom of the orifice is rather greater than that at the top of the orifice; but if the depth and breadth of the orifice be but small compared with its depth below the surface of the dam, we may take the depth of the centre of the hole for the mean depth, without any sensible error.

Newton concluded that the real velocity was less than the theoretical or computed velocity, in the ratio of 1 to $\sqrt{2}$; Abbé Bossuet as 100 to 150; and Michelotti as 5 to 8. Mr. Banks gives the experiments of these and others as follow:—

Newton......	·707—that is, $\frac{707}{1000}$ part of the computed [velocity.
Bossuet......	·615
Banks	·750
Michelotti...	·625
Helsham ...	·705
Smeaton ...	·631
	6)4·033
	·672 = mean.

The mean velocity of these experiments is $\frac{672}{1000}$ of the computed velocity, or their ratio is as $5\frac{3}{8}$ to 8, viz. at the depth of a foot the actual velocity from them is $5\frac{3}{8}$ or 5·4 feet, and in the same proportion for any other depth; for the computed velocity is $v = \sqrt{64\frac{1}{3}\,h} = 8\sqrt{h}$ nearly. Then, $·672 \times 8\sqrt{h} = 5·4\sqrt{h}$; and if $h = 1$ foot, it becomes 5·4. From this, Mr. Banks gives the following rule:—

Find the depth of the vessel in feet; then multiply the square root of that depth by 5·4, and the product will give the velocity in feet per second. This, multiplied by the area of the orifice in feet, gives the number of cubic feet which flows out in one second.

Ex.—If the orifice be made 9 feet below the surface of the water, the breadth of the orifice 3 feet, and length 4 inches, what quantity of water will flow out in one second?

$\sqrt{9} = 3$, and $3 \times 5·4 = 16·2 =$ the velocity per second.

4 inches is $\frac{1}{3}$ of a foot.

3 feet $\times \frac{1}{3} = 1$ foot, and $1 \times 16·2 = 16·2$ cubic feet, the water which flows out in a second.

To find the quantity of water discharged through notches or slits cut in the side of a vessel, the surface being with-

out motion, and if the velocity below varies as the square root of the depth, then the area of the perpendicular section will be a parabola, and may be found by the following rule:—

Rule.—Multiply the velocity at the bottom by the depth, and two-thirds of the product will give the area; and the area, multiplied by the breadth of the slit, gives the number of cubic feet discharged.

Ex.—If the depth of a notch or slit be 6 inches, and the breadth 8 inches, required the quantity which will be discharged in 20 seconds.

The depth in feet is ·5, the square root of which is ·707, and $5·4 \times ·707 = 3·8178$, two-thirds of which is 2·5452.

By the rule, $2·5452 \times ·5 = 1·2726$, which, multiplied by the breadth, which is $\frac{8}{12}$ of a foot, gives ·8484 feet per second; and $·8484 \times 20 = 16·9680$ cubic feet.

On Pumps.

For engines drawing water out of mines, it is necessary to have rules for finding the weight of the column of water which the engine has to lift.

The following is a simple rule, taken from Brewster's Ferguson:—

Rule.—Square the diameter of the pump in inches, and the product is the number of pounds avoirdupois of water contained in a yard's length of pipe; and one-tenth of the product will be the number of ale gallons in a yard's length of pipe.

Ex.—What weight of water will an engine have to lift, the diameter of the pump being 10 inches, and the length 30 fathoms?

$10 \times 10 = 100$ *lbs.* = the weight of water in one yard's length of pipe.

And since 30 fathoms is equal 60 yards, we have 100 *lbs.* $\times 60 = 6000$ *lbs.* = 2 *tons*, 13 *cwt.* 2 *qr.* 8 *lbs.*

Ex. 2.—What quantity of water can be discharged through a pipe 6 inches diameter, and 270 feet perpendicular height, at the rate of 240 feet per minute?

$\frac{270}{3} = 90$ yards, whole length of the pipe.

$6^2 \times 90 = 3240$ *lbs.* of water contained in the pipe.

And $\frac{240}{3} = 80$ yards.

$6^2 \times 80 = 2880$, and $\frac{2880}{10} = 288$ ale gallons per minute.

The above rule may be considered as near enough in practice; but if great nicety is required, the following rule should be used:—

Rule.—Multiply the area of the pump, in inches, by its length, also in inches; and the product gives the content in solid inches; and this, multiplied by the weight of one solid inch, gives the weight in pounds.

Ex.—What weight of water will a pump contain, the diameter of which is 10 inches, and the length 10 fathoms?

10 fathoms = 720 inches.

$10 \times 10 \times \cdot 7854 = 78 \cdot 54$ = area of the pump.

720 = length of pump in inches.

```
           15708
          54978
          ------
          56548·8 solid inches of water in the pump.
           ·03616 weight of one solid inch.
          -------
          3392928
         565488
        3392928
       1696464
       ----------
       2044·804608 = weight of water in pounds,
                   = 18 cwt. 1 qr.
```

ON RAIL-ROADS.

In entering on the subject of rail-roads, it will be necessary to give a short summary of experiments on friction, and an explanation of the different kinds of forces by which bodies are put in motion. A knowledge of these will prepare the mind for the reception of what has to follow.

*A short Summary of Experiments, selected from those of Vince, Coulomb, and others.**

Coulomb found, by a great number of very accurate experiments, that bodies sliding on a plane, from the action of a constant force, were uniformly accelerated.

These experiments shew that the retardation arising from friction is constant; for when a body is acted upon by a constant force, and is opposed by a resistance which is also uniform, the motion of the body will be continually accelerated.

Vince's experiments also shew, that if one hard body moves upon another, the friction will be an uniformly retarding force. Hence the quantity of friction does not

* To find the quantity of friction, let M = moving force considered as a weight; F = the quantity of friction considered as equivalent to a weight without inertia drawing the body back upon a horizontal plane; W = the weight of the body on the horizontal plane; and s = the space described by M in the time t. Then the whole accelerating force (the force of gravity being unity) will be—

$\frac{M - F}{M + W}$, and by Art. 109, Part I. $s = \frac{M - F}{M + W} \times 16\frac{1}{12}\, t^2$,

$$\therefore F = M - \frac{(M + W)\, s}{16\frac{1}{12}\, t^2}$$

depend on the velocity with which the rubbing parts move upon each other.

From the experiments made by Stephenson and Wood, it appears that the force required to keep a given weight in motion does not vary with the velocity: thus, a force of 14 *lbs.* was found to overcome friction, and keep in motion an empty coal-waggon, weighing 23·25 *cwt.* on a rail-road; and that on doubling the velocity, no more force was required. Further also it appears, that on increasing the weight or load, the power required to overcome the friction, and keep the waggon in motion, did not increase in similar proportion, but up to 76·25 *cwt.* was about one-fourteenth less.

Coulomb also found that the friction of wood sliding on wood became less when the body began to move than it had been the instant before, in the ratio nearly of 2 to 9. Its intensity afterwards did not change.

Friction can be diminished by making the surfaces smooth; but there is a limit, for the surfaces may receive such a polish that the attraction of cohesion may produce a sensible effect.

Also, friction may be diminished by rubbing the surfaces with some unctuous matter. Thus, Coulomb found that when an iron axle moved in brass, the friction was $\frac{1}{6}$; but when the axle was rubbed with clean tallow, the friction was $\frac{1}{11}$; and when oil of olives was employed, the friction was about $\frac{1}{8}$.

In woods, and also in metals, the frictions appear to be constant and independent of the velocity; but this is not the case where woods and metals rub against each other: here the friction is sensibly increased by an increase of velocity, but in a much less proportion.

The friction of axles varies according to the materials of which the axle and bush are made. If the axle be of iron, and the bush of brass, the friction is $\frac{1}{7}$ of the pressure; and when the bush and axle are both wood, there is

still less friction, for it then amounts to only $\frac{1}{13}$ of the pressure.

As before stated, it is necessary to make a distinction among the forces by which bodies are moved. Some forces act upon bodies with the same intensity, whether the bodies acted upon be at rest or in motion. Other forces act more powerfully on a body at rest, and become less as its velocity increases.

There is scarcely any subject less understood by the practical man than accelerating forces; for, in all the purposes of machinery, it is necessary to have uniformity of motion. If, however, a force act incessantly on a body with the same intensity, the motion of the body will be continually accelerated; for a single impulse will cause a body to move uniformly; but a continued succession of impulses will generate new velocities every instant of time, and will therefore accelerate the motion of the body.

But if a moving force is such, that when the motion begins, it is diminished in the intensity of its action, then the machine or body acted on will, after a short time, acquire an uniform velocity; and if the moving force be such that it acts with the same intensity on bodies in motion as at rest, and is opposed by a resistance which is uniform, the motion of the machine or body will be continually accelerated. But as a motion which is continually accelerated will not suit the purposes of machinery, it is necessary to have recourse to some contrivance which will render the motion uniform.

A state of uniform motion is sometimes produced in machines where gravity is the moving power, by means of a resistance so constituted as to increase with the velocity: when the velocity has reached a certain quantity, the retarding force becomes equal to the accelerating, and the velocity of the machine remains uniform.

This is exemplified in the motion of a jack: the resistance which the fly makes increases with the velocity, and,

after a certain time, prevents further acceleration. A pendulum in a clock is also used for this purpose.

But, again, if the power which acts upon a body be one that acts more forcibly upon a body at rest than in motion, its action is of course diminished, and the acceleration in the second instant is less than in the first. This diminution of the accelerating force continues to increase till the instant when that force becomes equal to the resistance, or till the velocity generated every instant by the one be just equal to the velocity destroyed every instant by the other. The acceleration then ceases, and the motion of the body becomes uniform.

This latter kind of force is exhibited by the strength of men and of all animals; for their strength is most powerful when exerted against a resistance which is at rest. When the resistance is overcome, and the animal is in motion, its force is diminished; and, with a certain velocity, the animal can do no work at all, and can only keep up the motion of its own body.

Also, the elasticity of springs, and the impulse of fluids, are examples of the same kind; but gravity is a force which acts with the same intensity on a body in motion as on a body at rest.

Mr. Sylvester, in his valuable Report on Rail-roads, &c. has given principles similar to what have been here laid down; of which the following is an extract:—

"Agreeably to the principles laid down in the commencement, when a force is applied equal to the friction, the smallest force above that would, if continued, generate any required velocity. But it will be desirable to have such a force at command as will generate the necessary velocity in a short time, and when that has been accomplished, to reduce this force, but still to leave it fully equal to the friction. If any part of the route has an inclination, there ought to be an extra force at command, above what would be required for a dead level. The plane on which this experiment was made, inclined, in the direc-

tion of the load about $\frac{1}{5}$ of an inch to a yard. This is as great, or perhaps a greater, inclination than any rail-road ought to have, where loaded carriages go up and down. The moving force ought, therefore, to be always greater than the friction added to the force which is required to overcome the inclination of the plane. The latter force assists the body to go down, and equally resists it in moving upwards.

"On this account, I have used, or supposed, a moving force, which will give the velocity of 5 miles an hour, or $7\frac{1}{3}$ feet per second, in the space of one minute. This will be performed down the above plane by the engine making 45 strokes per minute (the circumference of the wheel being 9 feet), with a pressure of 9·7 *lbs.* upon an inch, of each of the two cylinders, the area of each being 63·6 square inches. The weight of the engine and 16 waggons is equal to 154,560 *lbs.* or nearly 70 tons. The velocity of 5 miles an hour being acquired after one minute, the only force to keep the whole in motion, at the same rate, will be the difference between the gravity of the weight down the plane and the friction. The friction is 900 *lbs.*; the gravitating force of the weights down the plane 540 *lbs.*; therefore 900 — 540 = 360 *lbs.*

"If the same weight, at that speed, had to move on a dead level, and acquired the same velocity in one minute as before, the moving force would require to be 1781 *lbs.* which would require a pressure of 13·7 *lbs.* upon one inch. But after the speed is obtained, it will require only 7 *lbs.* to keep it moving at the same rate. If the same load were required to move up the plane, it would require a moving force of 2328 *lbs.* or a pressure upon every square inch of 18·3 *lbs.* And this velocity would be kept up by a constant pressure of 1447 *lbs.* which will be 11·3 *lbs.* upon every inch of the piston.

"In starting the engine, in the first instance, and giving the required velocity, it is probable the effects will agree very nearly with these calculations; namely, 154,560 *lbs.*

moved at the rate of 5 miles an hour, with a pressure of 9·7 *lbs.* upon every inch of the piston. Whether the pressure were reduced to the difference between the friction and the force upon the plane, which is calculated at 2·8 *lbs.* it is difficult to say, as there was no steam-gauge to indicate the pressure when the engine was going."

Some very useful articles on rail-roads have appeared in the Scotsman newspaper, and in the Manchester Guardian. The Scotsman, after having gone through various theoretical expositions, continues:—

"Having developed the theory of the motion of carriages on horizontal railways, we shall have little more to do with mathematical discussions, and shall now turn our attention to points of a practical nature, better adapted to the taste of ordinary readers. But, first, we shall bring under the eye again, the effect of a given quantity of power on a railway, and on a canal, in a calm atmosphere; for it is only in a calm atmosphere that the results can be properly compared.

"We have found that a *boat* weighing with its load 15 tons, *and a waggon of the same weight*, the one on a canal, and the other on a railway, would be impelled at the following rates, by the following quantities of power (which we have stated both in pounds and in horse power), reckoning one horse power equal to 180 pounds.

	Boat on a Canal.		*Waggon on a Railway.*	
Miles per Hour.	Power in Pounds.	Horse Power.	Power in Pounds.	Horse Power.
2	33	$\frac{1}{5}$	100	$\frac{1}{2}$
4	133	$\frac{2}{3}$	102	$\frac{1}{2}$
6	300	$1\frac{3}{4}$	105	$\frac{1}{2}$
8	533	3	109	$\frac{1}{2}$
12	1200	7	120	$\frac{2}{3}$
16	2133	12	137	$\frac{3}{4}$
20	3325	18	158	1

Strictures on the Scotsman, from the Manchester Guardian.

"The object of the papers on rail-roads which appeared in the Scotsman, was, in a great measure, to shew the practicability of transporting commodities upon rail-roads at a very considerable speed; and (with some fallacies, which we shall endeavour to point out) they contain a great deal of valuable information, on the relative merits of highways, canals, and rail-roads. The principal point, however, and the one to which we shall confine our observations, is an enunciation of the laws which regulate the friction of rolling and sliding bodies, as deduced from the experiments of Vince and Coulomb. With a view to the illustration of this part of the subject, some very important and conclusive experiments have recently been made in this town, to which we shall by and by have occasion to refer at some length; but, before doing so, we must make a few observations on the rule laid down by the Scotsman, and the misconceptions which appear to have prevailed respecting it, both in that journal and in other quarters.

"After comparing the resistance experienced by a boat moving through the water, with the friction which retards the progress of a waggon on a rail-road, and stating that they are governed by different laws, the Scotsman notices the conclusions established by the experiments of Vince and Coulomb; the most important of which is, that the *friction of rolling and sliding bodies is the same for all velocities*. The writer then observes:—

"'It is with this last law only that we have to do at present; and it is remarkable that the extraordinary results to which it leads have been, as far as we know, entirely overlooked by writers on roads and railways. These results, indeed, have an appearance so paradoxical, that they will shock the faith of practical men, though the principle from which they flow is admitted without question by all scientific mechanicians.

"'First, It flows from this law, that (abstracting the resistance of the air) if a car were set in motion on a level railway, with a constant force greater in any degree than is required to overcome its friction, the car would proceed with a motion continually accelerated, like a falling body acted upon by the force of gravitation; and however small the original velocity might be, it would in time increase beyond any assignable limit. It is only the resistance of the air (increasing as the square of the velocity) that prevents this indefinite acceleration, and ultimately renders the motion uniform.

"'Secondly, Setting aside again the resistance of the air (the effects of which we shall estimate by and by), the very same amount of constant force which impels a car on a railway at two miles an hour, would impel it at ten or twenty miles an hour, if an extra force were employed at first to overcome the *inertia* of the car, and generate the required velocity. Startling as this proposition may appear, it is an indisputable and necessary consequence of the laws of friction.

"'Now, it would at all times be easy, as we shall afterwards show, to convert this accelerated motion into a uniform of any determinate velocity; and from the nature of the resistance, a high velocity would cost almost as little, and be as readily obtained, as a low one. For all velocities, therefore, above four or five miles an hour, railways will afford facilities for communication prodigiously superior to canals, or arms of the sea.'

"Now we are perfectly satisfied, both by the experiments of Vince and Coulomb, and those more recent and conclusive experiments to which we have already alluded, that the rule laid down here is correct; but the writer ought to have guarded against the misconception to which his last paragraph is liable. When he says that a high velocity would cost almost as little as a low one, he should have said that it would cost as little per mile, or as little over any given space: for it cannot be his meaning, that a

carriage can be kept moving for an hour, or for any given time, at a high velocity, with as little expenditure of power as at a low velocity. Yet this he has been generally understood to mean, and a great deal has been written and said with a view to prove that he was mistaken; when in fact he was only misunderstood. In a subsequent article, however, the author appears, in some degree, to have fallen into the same error into which he has led other persons. He says:—

"'Every body knows that the rate of stage-coach travelling in this country has increased within the last twenty-five years, from six or seven miles an hour to eight or nine, and this, too, before roads were M'Adamized, and with much less injury to the horses than was anticipated. Supposing that a coach-horse could run fourteen miles unloaded, with the same muscular exertion which carries forward the stage-coach at eight or nine miles, then Professor Leslie's formula becomes 3-4ths $(14 - v)^2$.* Each horse would, of course, draw with a force of 48 *lbs.* at six miles, and of 27 *lbs.* at eight miles an hour. But if the friction increased in the ratio of the velocity, the load upon each horse would increase from 48 *lbs.* to 60 *lbs.* when the speed increased from six to eight miles an hour: and as the horse exerting the same strength would only pull with a force of 27 *lbs.* he would thus have more than double work to do, which is plainly impossible. But admit that the friction is equal in equal times; then, since the time is diminished 1-4th by increasing the speed from six to eight miles an hour, the horses have actually 4-5ths less to do; the load upon each is reduced from 48 *lbs.* to

* Professor Leslie's formula is $(12 - v)^2$ where v denotes the velocity in miles an hour. "Thus, if a horse beginning his pull with the force of 144 *lbs.* would draw 100 *lbs.* at a walk of two miles an hour, but only 64 *lbs.* when advancing at double that rate, and not more than 36 *lbs.* if he quickened that pace to six miles an hour, his greatest performance would hence be made with the velocity of four miles an hour."

36, and the horse would have to increase its exertion only 1-3rd, that is, from 27 *lbs.* to 36. The facts, we believe, will be found strictly consistent with this hypothesis, and decidedly at variance with the other. However strange it may sound, then, to common observers, it is practically true, that a smaller absolute amount of force will drag a coach over the same space in three hours than in four, and in one than in two.'

"This paragraph seems to us to contain a very obvious fallacy. If the speed be increased from six miles an hour to eight, the horses have by no means 1-4th less work to do, supposing the friction a constant quantity, and the traction consequently the same. It is true that they exert this power for a shorter time, but it is over the same distance. Supposing the power of traction necessary to overcome the friction is 1000 *lbs.* then that power must be extended over every yard of the distance, whether the carriage moves at six or eight miles an hour: and it is by the distance, not the time, that the power must be measured."

On the Friction of Carriages.

The friction of carriages must be determined by experiments; and the best sources of information appear to be the works of Mr. Tredgold and Mr. Wood; but there is a great disparity between them. Mr. Tredgold proceeds in a manner similar to the following:—

Let P = the moving power, W = the weight of the carriage and its load, F = the resistance from friction at the axle, i = the angle of the plane's inclination, R = radius of the wheels, r = radius of the axles, and f = the friction when the pressure is unity. Then, $P - F$ = the power which is employed in drawing the carriage up the plane; and, by Art. 116, Part I. we have $P - F = W \times \frac{h}{l} = W \times \sin. i$.

$$\therefore P = W \, sin. \, i + F.$$

But F is proportional to the pressure produced on the axles from the action of these forces; and, by Art. 66, Part I. these forces are as the perpendicular height of the plane and the base respectively. Therefore, the force in the direction parallel to the base $= W \, cos. \, i$, and the force in the direction of the perpendicular is $W \, sin. \, i$. The pressure on the axis, which is the resultant of these forces $=$ $\sqrt{\left\{ W^2 \, (cos.^2 i + sin.^2 i) \right\}}$ but $cos.^2 i + sin.^2 i = 1$.

$$\therefore \sqrt{\left\{ W^2 \, (cos.^2 i + sin.^2 i) \right\}} = \sqrt{W^2} = W.$$

Therefore, $F = \frac{W f r}{R}$; consequently, $P = W \, sin. \, i + \frac{W f r}{R} = W \left(sin. \, i + \frac{f r}{R} \right)$ = the power to move a carriage up an inclined plane.

And $W \left(sin. \, i - \frac{f r}{R} \right)$ = the tendency of the carriage to descend, which becomes 0 when $sin. \, i = \frac{f r}{R}$. Also when $sin. \, i$ is less than $\frac{f r}{R}$, we have $P = W \left(\frac{f r}{R} - sin. \, i \right) =$ the power to move a carriage down an inclined plane.

In this latter equation, when $\frac{f r}{R} = sin. \, i$, we have $P =$ 0, or the carriage will descend by its own weight. Hence we have an easy experimental mode of arriving at the friction by adjusting the inclination of the rails till the carriage barely moves upon them, and then the friction is expressed by the sine of the plane's inclination.

Again, when the plane of the rails is level, $sin. \, i = 0$.

$$\therefore P = \frac{W f r}{R}$$

which gives the power equal to the resistance from friction.

Mr. Tredgold, at pages 52 and 53, gives the result of the experiments, which he tried with a carriage the wheels of which were 8 inches diameter.

He increased the inclination of the rails till the carriage would descend with a continuous motion when loaded with 56 *lbs.* the weight of the carriage and wheels being $25\frac{3}{4}$ *lbs.* The motion was perceptibly slower when 86 *lbs.* more were added; but it was regular as before; and when the whole load was taken off, the carriage ran very freely, but at a rate not exceeding an average velocity of one foot per second; a space of 9 feet being described in from 9 to 10 seconds. The inclination of the plane was found to be 1·76 inches in a distance of 12 feet. This gives $\frac{1}{81}$ of the weight nearly for the moving force.

The motion was not so regularly accelerated as to allow of comparison to the laws of accelerated motion.

In the next experiment which he tried, the wheels were removed from the carriage, and 4-inch wheels were put on. When the rails had the same inclination as in the preceding experiment, the carriage would not move without additional force. The inclination was increased to 3·35 inches in 12 feet, measured on the rail, before the carriage would move with the same velocity as in the preceding trial; consequently, the force was equivalent to $\frac{1}{43}$ part of the weight. When 136 *lbs.* were added to the weight of the carriage, the motion was slower. The weight of the carriage, with the small wheels to it, was $17\frac{1}{2}$ *lbs.* From these experiments, it appears that the resistance does not decrease exactly one-half by doubling the size of the wheels; but the difference is not more than $\frac{1}{18}$ from that ratio.

The inclination was increased till the sine of the angle was ·039, and the time of descent was counted by the beat of a seconds pendulum, the space described being increased or diminished until the carriage struck against the block at the lower end of the rails at an even second. With the

4-inch wheels on, and a load of 40 *lbs.* the carriage described 8·9 feet in 5 seconds and 3·3 feet in 3 seconds; therefore the spaces described are very nearly as the squares of the times.

With 8-inch wheels and a load of 40 *lbs.* the carriage described 7·7 feet in 4 seconds.

We have shewn that W $\left(sin.\ i - \frac{f\,r}{R}\right)$ is equal to the force which accelerates a carriage down an inclined plane; and by Formula, Part I. page 83, the force which accelerates a body down an inclined plane $= \frac{s}{16\frac{1}{12}\,t^2}$; but we may take $\frac{s}{16\,t^2}$ as sufficiently near in practice.

$$\therefore\ sin.\ i - \frac{f\,r}{R} = \frac{s}{16\,t^2}$$

$$\therefore \frac{f\,r}{R} = sin.\ i - \frac{s}{16\,t^2}$$

In the first of the latter experiments, $sin.\ i = \cdot 039$, $s =$ 8·9 feet, and $t = 5$ seconds; if these be substituted in the above equation, we have

$$\frac{f\,r}{R} = \cdot 039 - \frac{8 \cdot 9}{16 \times 25} = \cdot 0166 = \frac{1}{60}$$

The second trial, where $s = 3{\cdot}3$ feet, and $t = 3$ seconds, gives $\frac{f\,r}{R} = \cdot 016 = \frac{1}{62 \cdot 5}$

In the last, $s = 7{\cdot}7$ feet, and $t = 4$ seconds, hence

$$\frac{f\,r}{R} = \cdot 039 - \frac{7 \cdot 7}{16 \times 16} = \cdot 0089 = \frac{1}{112} \text{ nearly.}$$

The ratio of the friction to the pressure, as deduced from these experiments, is the next subject to be considered; and since the diameter of the axis is ·55, and the diameter of the wheel 4 inches, when the whole resistance is $\frac{1}{60}$, we have $\frac{f\,r}{R} = \frac{f \times \cdot 55}{4}$; but $\frac{f\,r}{R} = \frac{1}{60}$.

$$\therefore \frac{f \times \cdot 55}{4} = \frac{1}{60}, \text{ or } f = \frac{1}{8 \cdot 25} = 0{\cdot}1212.$$

From the experiments with the 8-inch wheels, we have

$$\frac{f \times \cdot 55}{8} = \frac{1}{112}, \text{ or } f = \frac{1}{7\cdot 7}$$

The mean between these is nearly $\frac{1}{8}$, and this will be about the friction in practical cases.

Mr. Grimshaw, of Sunderland, made a great many experiments on the friction of wheel-carriages. He laid a cast-iron railway down upon beams of wood, and placed upon this railway the carriages which he used in conveying coals down to the river. He then elevated those beams at one end until they formed different angles with the horizon, and observed the time the carriages were in descending from one end to the other when the plane was elevated to different angles; and by comparing the spaces actually passed over by the carriage with the space which gravity would have caused the body to describe in the same time when descending freely, the amount of retardation caused by the friction was ascertained.

The result was as follows:—

Loaded carriage weighing altogether 8522 *lbs.* friction equal to 50 *lbs.* or the $\frac{1}{170}$th part of its weight.

Empty carriage weighing 2586 *lbs.* friction 10 *lbs.* or $\frac{1}{258}$th part of its weight.

Mr. Wood, of Killingworth colliery, has made a very extensive range of experiments, and with very great care and attention. They may, therefore, be relied on much better than those of any other writer on this subject. He states the friction to be $\frac{1}{200}$th part of the weight; but, considering the improvements that have been made in carriages, he says it may, in practice, be taken at the $\frac{1}{240}$th part of the weight.

The whole resistance is resolvable into two parts; that which acts upon the axles, and that which acts upon the rails.* Mr. Wood, by a great number of experiments,

* The friction at the rails is that of rolling; but the friction at the axles is that of rubbing, or attrition as it is sometimes called.

has determined the resistance of the rolling of the wheels to be $\frac{1}{1000}$th part of the whole weight. His experiments also shew that this ratio is not increased by an increase of the weight, and that it is nearly the same in velocities varying from 5·5 to 14·45 feet per second; so that the resistance by the rolling of the wheels is a uniformly retarding force, both with respect to velocity and weight.

Now, supposing the resistance from the wheels to be $\frac{1}{1000}$th part of the weight, and the whole amount of friction to be known, we can easily obtain that of the attrition at the axles; and the following table, which is taken from Mr. Wood's Treatise on Rail-roads, second edition, page 222, will give the results of his experiments:—

	Weight of carriage in lbs. including wheels and axles.	Weight of carriage resting on the axles.	Total resistance in lbs.	Resistance of wheels on rails, equal to the 1000th part of the weight in lbs.	Resistance of the axles by attrition, in lbs.	Resistance by attrition, in parts of the weight, in lbs.	Ratio of the diameter of the wheels, to that of the axles, the latter = 1.	Ratio of friction to insistent weight.
1	8540	7280	39·	8·54	30·46	239	12·36	19·
2	2604	1344	12·5	2·60	9·90	136	12·36	11·
3	2604	1344	13·5	2·60	10·90	123	12·36	10·
4	4816	3584	26·	4·81	21·19	169	12·36	13·6
5	7056	5824	34·	7·05	26·95	216	12·36	17·4
6	8512	7280	40·	8·51	31·49	231	12·36	19·
7	8456	7224	39.	8·45	30·55	236	12·36	19·
8	9408	8096	39·35	9·40	29·95	270	11·6	23·2
9	9408	8096	41·46	9·40	32·06	252	11·6	21·7
10	9408	8096	44·19	9·40	34·79	232	11·6	20·
11	3472	2160	12·73	3·47	9·26	233	11·6	20·
12	9100	7840	39·	9·10	29·90	262	12·36	21·2

From this table it appears that there is a very great difference in the amount of friction between the experiments of Mr. Tredgold and Mr. Wood; but, as we

have observed before, the great number of experiments which Mr. Wood has made, their coincidence, and the care and judgment with which they were performed, are particular recommendations in their favour. Indeed, Mr. Tredgold observes that he did not take the greatest care in preparing his model, for he considered that this could not be kept up in larger carriages; but in this he appears to be mistaken.

On Locomotive Engines.

Since the opening of the Liverpool and Manchester Railway, the most astonishing improvements have been made in the principles of these engines. Previous to that time, they were hardly capable of effecting a greater speed than six miles, and very frequently not more than four or five miles an hour: but now we find their regular day's work averaging more than 15 miles an hour; and, as will hereafter be seen, the Rocket has traversed the space of seven miles at the prodigious rate of 30 miles per hour, but without any train or tender; and allowance being made for stoppages, the seven miles may be considered as performed at the rate of 35 miles per hour.

A very valuable experiment was made by Mr. Wood, of Killingworth colliery. He caused five loaded waggons, weighing each 9408 *lbs.* to descend an inclined plane railway, rising 134 inches in 388 yards, or $\frac{1}{3}$ of an inch to a yard nearly. He found that the waggons passed over this space in 2 minutes. He then attached an engine to these waggons, weighing 9 tons, making the whole weight 67,200 *lbs.*; and he found that the time of the whole passing over the 388 yards was now $2\frac{1}{4}$ minutes. In this case, the moving force was the difference between the friction and the force of the weight down the plane. No steam was furnished from the boiler; and the force of the weight gave motion to the wheels of the engine, to the pistons, and all other parts connected with the same. By this

means, the friction of the wheels and all the other parts of the engine, as well as the friction of the five waggons, was employed to resist the force of the weight down the plane.

This gives the friction of the five waggons,

$F = \left(\frac{H}{L} - \frac{s}{g\,t^2}\right)W$, in which $W = 47040$ *lbs.* $H = 134$, $L = 13968$, $s = 1164$, $g = 16\frac{1}{12}$, and $t = 120$ seconds.

$$\therefore F = \left(\frac{134}{13968} - \frac{1164}{131600}\right) \times 47040 = 216 \textit{ lbs.}$$

And for the waggons and engines together,

$$F = \left(\frac{134}{13968} - \frac{1164}{361875}\right) \times 67200 = 428 \textit{ lbs.}$$

Then, $428 - 216 = 212$, the friction of the engine, including that of the wheels.

The resistance from the friction of the axles and the action of the wheels on the rails, being estimated at the 200th part of the weight, gives 100 *lbs.* which leaves 112 *lbs.* for the friction of the pistons, &c.

We will now give an account of the grand competition of locomotive engines at the Liverpool and Manchester Railway, selected from Mr. Wood's Treatise on Railroads and the Mechanic's Magazine.

Mr. Harrison, who was one of the directors, had been for some time of opinion that the excitement of a reward, publicly offered, would be the most likely mode of effecting their object; and in this opinion the other directors ultimately coincided. Accordingly, on the 20th of April, 1829, they resolved on offering a premium of £500 for the best locomotive engine, subjected to the following stipulations and conditions:—

"*Stipulations and Conditions on which the Directors of the Liverpool and Manchester Railway offer a Premium of £500 for the most improved Locomotive Engine.*

"1st, The said engine must 'effectually consume its own smoke,' according to the provisions of the Railway Act, 7 Geo. IV.

"2nd, The engine, if it weighs six tons, must be capable of drawing after it, day by day, on a well-constructed railway, on a level plane, a train of carriages of the gross weight of twenty tons, including the tender and water-tank, at the rate of ten miles per hour, with a pressure of steam on the boiler, not exceeding 50 *lbs.* per square inch.

"3rd, There must be two safety-valves, one of which must be completely out of the controul of the engine-man, and neither of which must be fastened down while the engine is working.

"4th, The engine and boiler must be supported on springs, and rest on six wheels; and the height, from the ground to the top of the chimney, must not exceed fifteen feet.

"5th, The weight of the machine, *with its complement of water* in the boiler, must, at most, not exceed six tons; and a machine of less weight will be preferred, if it draw *after* it a *proportionate weight*; and if the weight of the engine, &c. do not exceed *five tons*, then the gross weight to be drawn need not exceed fifteen tons, and in that proportion for machines of still smaller weight; provided that the engine, &c. shall still be on six wheels, unless the weight (as above) be reduced to four tons and a half, or under, in which case, the boiler, &c. may be placed on four wheels. And the Company shall be at liberty to put the boiler-fire, tube, cylinders, &c. to a test of a pressure of water not exceeding 150 *lbs.* per square inch, without being answerable for any damage the machine may receive in consequence.

"6th, There must be a mercurial gauge affixed to the machine, with index-rod, shewing the steam-pressure above 45 *lbs.* per square inch.

"7th, The engine to be delivered complete for trial at the Liverpool end of the railway, not later than the 1st of October next.

"8th, The price of the engine, which may be accepted, not to exceed £550 delivered on the railway; and any engine not approved to be taken back by the owner.

"N. B. The Railway Company will provide the *engine tender* with a supply of water and fuel for the experiment. The distance within the rails is four feet eight inches and a half."

The number of competitors was at first reported to be ten; but only five were entered on the official list, in which they were described as follows:—

No. 1. Messrs. Braithwaite and Ericson, of London, "The Novelty."

2. Mr. Hackworth, of Darlington, "The Sans Pareil."

3. Mr. Robert Stephenson, of Newcastle upon Tyne, "The Rocket."

4. Mr. Brandreth, of Liverpool, "The Cycloped;" worked by a horse.

5. Mr. Burstall, of Edinburgh, "The Perseverance."

The original stipulations of the directors containing no regulations as to the mode of trying the powers of the different engines, the judges determined, that in order to ascertain the comparative merit of each, they should be subjected to the following practical test. And in consequence, a card, containing the following regulations, was distributed to the different competitors.

"LIVERPOOL AND MANCHESTER RAILWAY.

"*The following is the ordeal we have decided each locomotive engine shall undergo in contending for the premium of £500 at Rainhill:—*

"The weight of the locomotive engine, with its full complement of water in the boiler, shall be ascertained at the weighing-machine, by eight o'clock in the morning, and the load assigned to it shall be three times the weight thereof. The water in the boiler shall be cold, and there shall be no fuel in the fire-place. As much fuel shall be

weighed, and as much water shall be measured and delivered into the tender-carriage as the owner of the engine may consider sufficient for the supply of the engine for a journey of thirty-five miles and one half. The fire in the boiler shall then be lighted, and the quantity of fuel consumed for getting up the steam shall be determined, and the time noted.

"The tender-carriage, with the fuel and water, shall be considered to be, and taken as part of the load assigned to the engine.

"Those engines that carry their own fuel and water, shall be allowed a proportionate deduction from their load, according to the weight of the engine.

"The engine, with the carriages attached to it, shall be run by hand up to the starting-post, and as soon as the steam is got up to 50 *lbs.* per square inch, the engine shall set out upon its journey.

"The distance the engine shall perform each trip shall be one mile and three-quarters each way, including one-eighth of a mile at each end for getting up the speed, and for stopping the train: by this means, the engine, with its load, will travel one and a half mile each way, at full speed.

"The engine shall make ten trips, which shall be equal to a journey of thirty-five miles, thirty miles whereof shall be performed at full speed, and the average rate of travelling shall not be less than ten miles per hour.

"As soon as the engine has performed this task (which will be equal to the travelling from Liverpool to Manchester), there shall be a fresh supply of fuel and water delivered to her; and as soon as she can be got ready to set out again, she shall go to the starting post, and make ten trips more, which will be equal to the journey from Manchester, and back again to Liverpool.

"The time of performing every trip shall be accurately noted, as well as the time occupied in getting ready to set out on the second journey.

"Should the engine not be enabled to take along with it sufficient fuel and water for the journey of ten trips, the time occupied in taking in a fresh supply of fuel and water shall be considered, and taken as part of the time in performing the journey.

J. U. Rastrick, C. E. Stourbridge,
Nicholas Wood, C. E. Killingworth, } Judges.
John Kennedy, Manchester,

"*Liverpool, Oct. 6th*, 1829."

First Day, October 8.—The following is a correct account of the performance of the "Rocket" this day. The journey was 1½ mile each way, with an additional length of 220 yards at each end to stop the engine in, making in one journey 3½ miles. The first experiment was for 35 miles, which is exactly 10 journeys, and including all the stoppages at the ends, was performed in 3 hours and 10 minutes, being upwards of 11 miles an hour. After this, a fresh supply of water was taken in, which occupied 16 minutes; when the engine again started, and ran 35 miles in 2 hours and 52 minutes, which is upwards of 12 miles an hour, including all stoppages. The speed of the engine with its load, when in full motion, was from 14 to 17 miles an hour; and had the whole distance been in one continued direction, there is no doubt but that the result would have been 15 miles an hour. The consumption of coke was very moderate, not exceeding half a ton in the whole 70 miles. At several parts of the journey the engine moved at the rate of 18 miles an hour. The weights of the engine and its load were as follows:—

	Tons.	cwt.	qr.	lb.
Engine	4	5	0	0
Tender, with water and coke	3	4	0	2
Two carriages loaded with stones	9	10	3	26
Whole mass in motion	17	0	0	0

Mr. Hackworth's engine was withdrawn for the present.

Fifth Day, October 13.—It being understood that this was to be the day of a more decisive trial of Messrs. Braithwaite and Ericson's engine, there was almost as numerous an assemblage of spectators as on the first day of the competition. A fresh pipe had been substituted for the one which failed on the preceding trial; one or two other parts of the machinery, that were in a faulty state, had also been renovated; but the engine, with the exception of some of the flanges of the boiler being, as Mr. Ericson expressed it, rather *green*, was pronounced in a working state. The steam was on this occasion got up to a pressure of 50 *lbs.* in somewhat less than 40 minutes, and at an expenditure of about 15 *lbs.* of coke. The engine started to do the 70 miles for a continuance; but just as it had completed its second trip of three miles, when it was working at the rate of 15 miles an hour, the new cement of some of the flanges of the boiler yielded, and the engine was obliged to stop. Messrs. Braithwaite and Ericson, feeling that they could not get it into a working state in any reasonable time, retired from the contest, thus leaving Mr. Stephenson master of the field.

Sixth Day, October 14.—In the early part of the day, Mr. Stephenson's engine ascended the Rainhill inclined plane (which has an ascent of 1 in 96) several times with heavy loads of passengers, and did this at the rate of 12 miles an hour.

On Tuesday, the 20th of October, the judges appointed to report on the performances gave in their report to the directors; and in consequence of the opinion expressed by them, the prize of £500 was adjudged to Mr. Robert Stephenson, of Newcastle.

TABLE OF AREAS OF CIRCLES,

UP TO 80 INCHES.

DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.
1	·785	5	19·63	21	346·36	37	1075·21
1⅛	·994	5½	23·75	21½	363	37½	1104·4
1¼	1·227	6	28·27	22	380·13	38	1134·11
1⅜	1·484	6½	33·18	22½	397·6	38½	1164·15
1½	1·767	7	38·48	23	415·47	39	1194·59
1⅝	2·073	7½	44·17	23½	433·73	39½	1225·42
1¾	2·405	8	50·26	24	452·39	40	1256·64
1⅞	2·761	8½	56·74	24½	471·43	40½	1288·25
2	3·141	9	63·61	25	490·87	41	1320·25
2⅛	3·546	9½	70·88	25½	510·7	41½	1352·65
2¼	3·976	10	78·54	26	530·93	42	1385·44
2⅜	4·430	10½	86·59	26½	551·54	42½	1418·62
2½	4·908	11	95	27	572·55	43	1452·20
2⅝	5·411	11½	103·86	27½	593·95	43½	1486·17
2¾	5·939	12	113·	28	615·75	44	1520·53
2⅞	6·491	12½	122·71	28½	637·94	44½	1555·28
3	7·068	13	132·73	29	660·52	45	1590·43
3⅛	7·669	13½	143·13	29½	683·49	45½	1625·09
3¼	8·295	14	153·93	30	706·86	46	1661·9
3⅜	8·946	14½	165·13	30½	730·61	46½	1698·23
3½	9·621	15	176·71	31	754·76	47	1734·94
3⅝	10·32	15½	188·69	31½	779·31	47½	1772·05
3¾	11·04	16	201	32	804·29	48	1809·56
3⅞	11·79	16½	213·82	32½	829·57	48½	1847·45
4	12·56	17	226·98	33	855·3	49	1885·74
4⅛	13·36	17½	240·52	33½	881·41	49½	1924·42
4¼	14·18	18	254·46	34	907.92	50	1963·5
4⅜	15·03	18½	268·8	34½	934·82	50½	2002·96
4½	15·90	19	283·52	35	962·11	51	2042·82
4⅝	16·80	19½	298.64	35½	989·8	51½	2083·07
4¾	17·72	20	314·16	36	1017·87	52	2123·72
4⅞	18·66	20½	330	36½	1046·34	52½	2164·75

DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.	DIAMETER IN INCHES.	AREA IN INCHES.
53	2206·18	60	2827·44	67	3525·66	74	4300·85
53½	2248·01	60½	2874·56	67½	3578·47	74½	4359·16
54	2290·22	61	2922·47	68	3631·68	75	4417·87
54½	2332·83	61½	2970·38	68½	3685·29	75½	4476·97
55	2375·83	62	3019·07	69	3739·29	76	4536·47
55½	2419·22	62½	3067·96	69½	3793·67	76½	4596·25
56	2463·01	63	3117·25	70	3848·46	77	4656·63
56½	2507·19	63½	3166·92	70½	3903·60	77½	4717.31
57	2551·76	64	3217	71	3959·20	78	4778·37
57½	2596·53	64½	3267·46	71½	4015·16	78½	4839·73
58	2642·08	65	3318·31	72	4071·51	79	4901·68
58½	2687·83	65½	3369·56	72½	4128·25	79½	4963·92
59	2733·97	66	3421·20	73	4185·39	80	5026·56
59½	2780·31	66½	3473·23	73½	4242·13		

PARALLEL MOTION TABLES,

When the Length of the Stroke is not taken into Consideration.

Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.
72	18	158·111	78	27	96.5
—	21	124	—	30	76·8
—	24	96	—	33	61·36
—	27	75	—	36	49·111
—	30	58·8	—	39	39
—	33	46	—	42	30·857
—	36	36	—	45	24.2
—	39	27·9	—	48	18·75
—	42	21·428	—	51	14·3
—	45	16·2	—	54	10·666
—	48	12	—	57	7·7
—	51	8·6	—	60	5·4
—	54	6	84	18	242
78	18	200	—	21	189
—	21	154·7	—	24	150
—	24	121·5	—	27	120·3

Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.
84	30	97·2	96	57	26·7
—	33	79	—	60	21·6
—	36	64	—	63	17·3
—	39	52	—	66	13·636
—	42	42	—	69	10·5
—	45	33·8	—	72	8
—	48	27	—	75	5·7
—	51	21·35	102	30	172·8
—	54	16·666	—	33	144·3
—	57	12·8	—	36	121
—	60	9·6	—	39	101·7
—	63	7	—	42	85·738
—	66	4·9	—	45	72·2
90	18	288	—	48	60·75
—	21	226·7	—	51	51
—	24	181·5	—	54	42.666
—	27	147	—	57	35·5
—	30	120	—	60	29·4
—	33	98·4	—	63	24·14
—	36	81	—	66	19·636
—	39	66·7	—	69	15·8
—	42	54·857	—	72	12·5
—	45	45	108	36	144
—	48	36·75	—	39	122
—	51	30	—	42	103·716
—	54	24	—	45	88·2
—	57	19	—	48	75
—	60	15	—	51	63·7
—	63	11·5	—	54	54
—	66	8·727	—	57	45·6
—	69	6·4	—	60	38·4
—	72	4·5	—	63	32·14
96	24	216	—	66	26·727
—	27	176·3	—	69	22
—	30	145·2	—	72	18
—	33	120·3	—	75	14·52
—	36	100	114	42	123·428
—	39	83·3	—	45	105·8
—	42	96·428	—	48	90·75
—	45	57·8	—	51	77·8
—	48	48	—	54	66·666
—	51	39·7	—	57	57
—	54	32·666	—	60	48·6

Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of parallel Bar in Inches.	Length of Radius Rod in Inches.
114	63	41·3	132	72	50
—	66	34·9	—	75	43·32
—	69	29·3	—	78	37·384
—	72	24·5	—	81	32·111
—	75	20·3	—	84	27·428
120	42	146·761	—	87	23·27
—	45	125	—	90	19·6
—	48	108	—	93	16·35
—	51	93·35	138	48	168·75
—	54	80·666	—	51	148·4
—	57	69·6	—	54	130·666
—	60	60	—	57	115
—	63	51·5	—	60	101·4
—	66	44·181	—	63	89·3
—	69	37·7	—	66	78·545
—	72	32	—	69	69
—	75	27	—	72	60·5
—	78	22·564	—	75	52·9
—	81	18·77	—	78	46·153
126	42	168	—	81	40
—	45	145·8	—	84	34·643
—	48	126·75	—	87	30
—	51	110·3	—	90	25·6
—	54	96	—	93	21·7
—	57	83·5	144	48	192
—	60	72·433	—	51	169·6
—	63	63	—	54	150
—	66	54·545	—	57	182·7
—	69	47	—	60	117·6
—	72	39·25	—	63	104·14
—	75	34·7	—	66	92·181
—	78	29·33	—	69	89·3
—	81	25	—	72	72
—	84	21	—	75	63·5
—	87	17·48	—	78	55·846
132	48	147	—	81	49
—	51	128·6	—	84	42·857
—	54	112·666	—	87	37·35
—	57	98·7	—	90	32·4
—	60	86·4	—	93	28
—	63	75·5	—	96	24
—	66	66	—	99	20·4
—	69	57·5			

TABLES OF SAFETY VALVE LEVERS.

1. Diameter of the valve 4 inches, weight of the valve, &c. 3 *lbs.* length of the lever 24 inches, the distance between the fulcrum and valve 4 inches, and the weight of the lever 8 *lbs.*; then the weight put on at the end of the lever, to give 60 *lbs.* per square inch upon the valve, will be 58·332 *lbs.* or 58 *lbs.* 5⅓ *oz.* nearly.

For 10 *lbs.* pressure, distance from fulcrum 6·765 inches.
20 *lbs.* ....15·382 inches.
30 *lbs.* ....24 inches.

2. Length of the lever 16 inches, distance between the fulcrum and valve 2 inches, diameter of valve 2 inches, the weight of the lever 4 *lbs.* and weight of valve ½ *lb.*; it will require a weight of 9·7185 *lbs.* to be put on at the end of the lever to give 30 *lbs.* per square inch upon the valve; and the distances in inches which the weight must be from the fulcrum to give 10, 15, 20, 25, and 30 *lbs.* are respectively as follow:

10 *lbs.*	15 *lbs.*	20 *lbs.*	25 *lbs.*	30 *lbs.*
3·068	6·301	9·534	12·767	16

If the weight be taken off from the lever, then the weight on the valve from the action of the lever alone will give 5¼ *lbs.* per square inch.

Note, 9·7185 *lbs.* is 9 *lbs.* 11½ *oz.*

3. Weight of lever 4 *lbs.* and weight of valve, &c. 1 *lb.*; whole length of the lever 24 inches, distance between the fulcrum and valve 3 inches, diameter of valve 3 inches, weight put on at the end 42·05375 *lbs.* Distances from the fulcrum in inches:

10 *lbs.*	20 *lbs.*	30 *lbs.*	40 *lbs.*	50 *lbs.*
3·8292	8·8719	13·9146	18·9573	24

Table of the Weight of one Foot in Length of square Iron, from ¼ Inch to 6 Inches square; together with an Example to shew how the Table is calculated.

SIDE OF SQUARE.	WEIGHT.	SIDE OF SQUARE.	WEIGHT.	SIDE OF SQUARE.	WEIGHT.	SIDE OF SQUARE.	WEIGHT.
Inch	*lb.*	*Inch*	*lb.*	*Inch*	*lb.*	*Inch*	*lb.*
$\frac{1}{4}$	·208	$1\frac{3}{4}$	10·2	$3\frac{1}{4}$	35·2	$4\frac{3}{4}$	75·2
$\frac{3}{8}$	·468	$1\frac{7}{8}$	11·71	$3\frac{3}{8}$	37·96	$4\frac{7}{8}$	79·21
$\frac{1}{2}$	·833	2	13·33	$3\frac{1}{2}$	40·8	5	83·33
$\frac{5}{8}$	1·3	$2\frac{1}{8}$	15·05	$3\frac{5}{8}$	43·8	$5\frac{1}{8}$	87·55
$\frac{3}{4}$	1·87	$2\frac{1}{4}$	16·87	$3\frac{3}{4}$	46·87	$5\frac{1}{4}$	91·87
$\frac{7}{8}$	2·55	$2\frac{3}{8}$	18·80	$3\frac{7}{8}$	50·05	$5\frac{3}{8}$	96·30
1	3·33	$2\frac{1}{2}$	20·82	4	53·33	$5\frac{1}{2}$	100·80
$1\frac{1}{8}$	4·21	$2\frac{5}{8}$	22·96	$4\frac{1}{8}$	56.71	$5\frac{5}{8}$	105·47
$1\frac{1}{4}$	5·2	$2\frac{3}{4}$	25·2	$4\frac{1}{4}$	60·2	$5\frac{3}{4}$	110·21
$1\frac{3}{8}$	6·3	$2\frac{7}{8}$	27·55	$4\frac{3}{8}$	63·8	$5\frac{7}{8}$	115·05
$1\frac{1}{2}$	7·5	3	30·	$4\frac{1}{2}$	67·5	6	120
$1\frac{5}{8}$	8·8	$3\frac{1}{8}$	32·55	$4\frac{5}{8}$	71·3	.	

Ex.—What is the weight of a bar of iron $5\frac{1}{4}$ inches square?

```
     5·25 = side of the square.
     5·25
     ----
     2625
    1050
   2625
   -------
   27·5625 = area of the end of the bar.
        12 = length of the bar.
  --------
  330·75   = solid inches contained in the bar.
  ·27778   = the weight of one solid inch in parts of a
  ------      pound.
  264600
 231525
231525
231525
66150
-----------
91·8757350 = the weight of the bar in pounds.
```

Table of the Weight of one Foot in Length of round Iron, from $\frac{1}{4}$ Inch to 6 Inches Diameter; also an Example to shew how the Table is calculated.

DIAM.	WEIGHT.	DIAM.	WEIGHT.	DIAM.	WEIGHT.	DIAM.	WEIGHT.
Inch	*lb.*	*Inch*	*lb.*	*Inch*	*lb.*	*Inch*	*lb.*
$\frac{1}{4}$	·163	$1\frac{3}{4}$	8·01	$3\frac{1}{4}$	27·65	$4\frac{3}{4}$	59·06
$\frac{3}{8}$	·368	$1\frac{7}{8}$	9·2	$3\frac{3}{8}$	29·82	$4\frac{7}{8}$	62·21
$\frac{1}{2}$	·654	2	10·47	$3\frac{1}{2}$	32·07	5	65·45
$\frac{5}{8}$	1·02	$2\frac{1}{8}$	11·82	$3\frac{5}{8}$	34·4	$5\frac{1}{8}$	68·76
$\frac{3}{4}$	1.47	$2\frac{1}{4}$	13.25	$3\frac{3}{4}$	36·81	$5\frac{1}{4}$	72·16
$\frac{7}{8}$	2	$2\frac{3}{8}$	14·76	$3\frac{7}{8}$	39·31	$5\frac{3}{8}$	75·63
1	2·61	$2\frac{1}{2}$	16·36	4	41·89	$5\frac{1}{2}$	79·19
$1\frac{1}{8}$	3·31	$2\frac{5}{8}$	18·03	$4\frac{1}{8}$	44·54	$5\frac{5}{8}$	82·83
$1\frac{1}{4}$	4·09	$2\frac{3}{4}$	19·79	$4\frac{1}{4}$	47·28	$5\frac{3}{4}$	86·56
$1\frac{3}{8}$	4·94	$2\frac{7}{8}$	21·63	$4\frac{3}{8}$	50·11	$5\frac{7}{8}$	90·36
$1\frac{1}{2}$	5·89	3	23·56	$4\frac{1}{2}$	53.01	6	94·25
$1\frac{5}{8}$	6·91	$3\frac{1}{8}$	25·56	$4\frac{5}{8}$	56		

Ex.—What is the weight of a round bar of iron, $2\frac{1}{8}$ inches diameter and one foot long?

$$2\tfrac{1}{8} = 2{\cdot}125 = \text{diameter.}$$

```
   2·125
   2·125
   -----
   10625
   4250
  2125
 4250
 --------
 4·515625
    ·7854
 ----------
   18062500
  22578125
 36125000
31609375
-----------
3·546571875 = area of the end.
```

```
      3·546571875
               12 = length.
   ──────────────
     42·558862500 = solid inches in the bar.
           ·27778 = weight of one solid inch.
     ────────────
       3404709000
      2979120375
     2979120375
    2979120375
    851177250
    ─────────────
 11·822000825250 = the weight in pounds.
```

TABLE OF THE

WEIGHT OF CAST IRON PIPES,

12 INCHES LONG, FROM $\frac{1}{4}$ TO 1 INCH THICK.

Diam. of Bore	$\frac{1}{4}$ Inch.	$\frac{3}{8}$ Inch.	$\frac{1}{2}$ Inch.	$\frac{5}{8}$ Inch.	$\frac{3}{4}$ Inch.	$\frac{7}{8}$ Inch.	1 Inch.
Inch	lb.	lb.	lb.	lb.	lb.	lb.	lb.
1	3·06	5·06	7·36	9·97	12·89	16·11	19·63
$1\frac{1}{4}$	3·68	5·98	8·59	11·51	14·73	18·25	22·09
$1\frac{1}{2}$	4·29	6·9	9·82	13·04	16·56	20·4	24·54
$1\frac{3}{4}$	4·91	7·83	11·05	14·57	18·41	22·55	27
2	5·53	8·75	12·27	16·11	20·25	24·7	29·45
$2\frac{1}{4}$	6·14	9·66	13·5	17·64	22·09	26·84	31·85
$2\frac{1}{2}$	6·74	10·58	14·72	19·17	23·92	28·93	34·36
$2\frac{3}{4}$	7·36	11·5	15·95	20·7	25·71	31·14	36·81
3	7·98	12·43	17·18	22·19	27·62	33·29	39·28
$3\frac{1}{4}$	8·59	13·34	18·35	23·78	29·45	35·44	41·72
$3\frac{1}{2}$	9·2	14·21	19·64	25·31	31·3	37·58	44·18
$3\frac{3}{4}$	9·76	15·19	20·86	26·85	33·18	39·73	46·63

Diam. of Bore	1/4 Inch.	3/8 Inch.	1/2 Inch.	5/8 Inch.	3/4 Inch.	7/8 Inch.	1 Inch.
Inch	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*
4	10·44	16·11	22·1	28·38	34·98	41·88	49·1
4¼	11·1	17·08	23·37	29·97	36·87	44·08	51·6
4½	11·66	17·94	24·54	31·44	38·65	46·17	54
4¾	12·27	18·87	25·77	32·98	40·5	48·32	56·45
5	12·88	19·78	26·99	34·51	42·33	50·46	59
5¼	13·5	20·71	28·23	36·05	44·18	52·62	61·36
5½	14·11	21·63	29·45	37·58	46·02	54·76	63·81
5¾	14·73	22·55	30·68	39·12	47·86	56·91	66·27
6	15·34	23·47	31·91	40·65	49·7	59·06	68·73
6¼	15·95	24·39	33·13	42·18	51·54	61·21	72
6½	16·57	25·31	34·36	43·72	53·39	63·36	73·41
6¾	17·18	26·23	35·59	45·26	55·23	65·28	76·1
7	17·79	27·15	36·82	46·79	56·84	67·65	78·53
7¼	18·41	28·08	38·05	48·1	58·91	69·79	81
7½	19·03	29	39·05	49·86	60·74	71·95	83·45
7¾	19·64	29·69	40·5	51·38	62·59	74·09	86
8	20·02	30·83	41·71	52·92	64·42	76·23	88·35
8¼	20·86	31·74	42·95	54·45	66·26	78·38	90·81
8½	21·69	32 9	44·4	56·21	68·33	80·76	93·49
8¾	22·09	33·59	45·4	57·52	69·95	82·68	95·72
9	22·71	34·52	46·64	59·07	71·8	84·84	98·18
9¼	23·31	35·43	47·86	60·59	73·63	86·97	100·63
9½	23·93	36·36	49·09	62·13	75·47	89·13	103·1
9¾	24·55	37·28	50·32	63·66	77·32	91·28	105·54
10	25·16	38·2	51·54	65·2	79·16	93·42	108
10¼	25·77	39·11	52·77	66·73	80·99	95·57	110·44
10½	26·38	40·04	54	68·26	82·84	97·71	113
10¾	27	40·96	55·22	69·8	84·67	99·86	115·35
11	27·62	41·88	56·46	71·33	86·52	102·01	117·81
11¼	28·22	42·8	57·67	72·86	88·35	104·15	120·26
11½	28·84	43·71	58·9	74·39	90·19	106·3	122·71
11¾	29·45	44·64	60·13	75·93	92·04	108·45	125·18
12	30.06	45·55	61·35	77·46	93·6	110·6	127·6

Table of the Weight of cast Iron Pipes, 12 *Inches long, from* 12½ *to* 20½ *Inches Diameter, and from* ½ *Inch to* 2 *Inches thick.*

Diam. of Bore	½ Inch.	¾ Inch.	⅞ Inch	1 Inch.	1⅛ Inch	1¼ Inch.	1½ Inch	1¾ Inch	2 Inch.
Inch	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*	*lb.*
12½	63·5	97.3	114	132	149	167	205	243	285
13	66	101	118	137	154	173·5	212	252	294
13½	68·4	104·8	122	141·5	160	179	219	260	304
14	71	108·2	126	146	165	185	227	269	314
14½	73·4	112.3	130	151	170	192	234	277	324
15	75·8	115·7	135	156	176	198	242	286	334
15½	78·1	119	139	161	181	204	250	295	344
16	80·7	123	143	166	187	211	257	303	355
16½	83·1	126·5	147	170	192	217	264	312	363
17	85·5	130	152	178·5	198	223	271	322	376
17½	87·8	133·5	157	180·5	203	229	278	330	383
18	90·5	137	161	185	209	235	285	338	393
18½	93	140·5	166	190	217	241	293	347	402
19	95·5	144·8	169	195	222	247	300	354	412
19½	97·8	148·5	174	200	227	253	307	363	422
20	100	152	178	205	233	259	315	372	432
20½	102·5	156	183	210	238	265	323	381	442

To find the Weight of cast Iron Pipes 1 *Foot long.*

Given the diameter of bore 12 inches, thickness of metal ½ inch, to find the weight of one foot in length.

$$
\begin{array}{r}
\cdot 7854 \\
144 \\
\hline
31416 \\
31416 \\
7854 \\
\hline
113\cdot 0976
\end{array}
$$

144 = square of the diameter.

113·0976 = area of the bore.

The thickness of metal being half an inch, the diameter of the bore must be increased one inch; therefore the diameter to the outside will be 13 inches.

```
     ·7854
       169 = square of the diameter.
    ------
     70686
    47124
    7854
   --------
  132·7326 = area of both bore and metal.
  113·0976
  --------
   19·6350 = area of the metal.
        12
  --------
  235·620 = solid inches in foot of length
   ·2604  = weight of one solid inch.
  --------
    94248
  141372
  47124
  ---------
61·355448 = weight of one foot in length.
```

Table of the Diameters of Piston Rods for High Pressure Engines, Pressure from 20 to 50 lbs. per square Inch.

Diam. of Cylinder.	*lbs.* Pressure per square Inch.			
	20	30	40	50
Inch	*Inch*	*Inch*	*Inch*	*Inch*
10	1·264	1·55	1·78	2
11	1·3904	1·7	1·96	2·2
12	1·5168	1·86	2·13	2·4
13	1·643	2·03	2·3	2·6
14	1·769	2·17	2·5	2·8
15	1·896	2·32	2·67	3
16	2·0224	2·48	2·85	3·2
17	2·148	2·635	3·026	3·4
18	2·275	2·79	3·204	3·6
19	2·4016	2·945	3·382	3·8
20	2·528	3·1	3·56	4

This table may be continued to any length; for the first column increases ·126 for each inch in the cylinder, the second column increases ·155, the third column increases ·178, and the last increases ·2 for each inch the cylinder is increased in diameter.

As an example, suppose that it is required to find the diameter of a piston rod, the diameter of the cylinder being 24 inches, and the pressure 50 *lbs.* per square inch.

In this case, the cylinder is increased 4 inches beyond the limits of the table; and $4 \times \cdot 2 = \cdot 8$.

The diameter of a piston rod for a cylinder 20 inches, and 50 *lbs.* per square inch, is 4 inches; therefore 4·8 inches is the diameter of a piston rod for a cylinder the diameter of which is 24 inches, and the pressure 50 *lbs.* per square inch.

This table is calculated at upwards of double the pressure.

Flanches.

Mr. Jamieson, founder to Messrs. Hawks, informs us that they generally allow the weight of one foot in length of pipe for two flanches; but if this should not be considered sufficiently exact, the following example will shew how to calculate them.

What is the weight of a flanch ½ inch thick, diameter to the outside 20 inches, and inside diameter 10 inches?

Here the breadth of the metal on each side is evidently 5 inches.

$20^2 \times \cdot 7854 = 314 \cdot 16 =$ whole area of both bore and metal.
$10^2 \times \cdot 7854 = 78 \cdot 54 =$ area of the bore.

$235 \cdot 62 =$ area of metal.
$\cdot 5 =$ thickness of the metal.

$117 \cdot 810 =$ solid inches of metal in the flanch.

And $117 \cdot 81 \times \cdot 2604 = 30 \cdot 677724$ *lbs.* = weight of the flanch.

ADDITIONS AND CORRECTIONS.

FORCES.—If a weight W be suspended at C, the end of the arm of a crane, (see Fig. 50, Part I.) required the pressure at the end D of the spur, and also the pressure at A against the upright.

By the property of the lever, $\frac{BC \times W}{DB}$ = the pressure at D in the direction D E, and this pressure is to that in the direction D A as D E to D A; that is,

$DE : DA :: \frac{BC \times W}{DB} : \frac{BC \times DA \times W}{DE \times DB}$ the pressure in D A. Again, $DA : AE$ or $DB :: \frac{BC \times DA \times W}{DE \times DB} : \frac{BC \times W}{DE} = \frac{BC \times W}{AB}$ the pressure against A in the direction E A.

Ex.—Let B C = 10 feet, D B = 4 feet, A B = D E = 3 feet; then $DA = \sqrt{(4^2 + 3^2)} = \sqrt{25} = 5$; and let the weight W be 2 tons. Then, $\frac{BC \times DA \times W}{DE \times DB} = \frac{10 \times 5 \times 2}{3 \times 4} = \frac{100}{12} = 8\frac{1}{3}$ tons for the pressure on the spur D A. And $\frac{BC \times W}{AB} = \frac{10 \times 2}{3} = 6\frac{2}{3}$ tons, the force tending to break the upright F B in A.

Page 22, line 20, for A F, read A C.
Page 42, line 31, for A, B, C, D, E, read A, B, C, D.
Page 71, the number of the article on compression is 92, which is omitted.
Page 73, line 3 from the bottom, for b^2 read l^2
Page 79, line 15, omit "from the earth's centre," &c.
Page 102, line 2 from the bottom, for x read ×.
Page 105, line 7, for d^2 read d.
Page 117, line 2 from bottom, for considers, read says.

Safety Valve Lever.—When the lever is not uniform, find its centre of gravity by balancing it upon a fine edge; then measure the distance from the fulcrum to the centre of gravity, and

divide this distance by the distance between the fulcrum and valve, and multiply the quotient by the whole weight of the lever, and this product will give the whole weight upon the valve from the weight of the lever.

Thus, suppose the length of a lever to be 12 inches, and weight 3 *lbs.* the distance between the fulcrum and valve 2 inches, and the distance between the fulcrum and centre of gravity 4 inches; then 4 divided by 2 = 2, and 2 multiplied by 3 = 6 *lbs.* the weight on the valve from the action of the lever.

Page 130, line 13 from bottom, for from, read for.

Parallel Motion.—At page 133 there is a rule formed from the algebraic process, which does not appear to hold good in those cases where twice the length of the parallel bar is greater than the radius of the beam. This rule will rather embarrass the practical man if he is not acquainted with algebra; but if he be acquainted with that science, it presents no difficulty.

When twice the length of the parallel bar is greater than the radius of the beam, the following rule must be used:—

Rule.—Subtract the radius of the beam from twice the length of the parallel bar, and multiply the remainder by the square of half the length of the stroke, for a dividend. Then, from the square of the radius of the beam subtract the square of half the length of the stroke, and extract the square root of the remainder; subtract this root from the radius of the beam, and multiply the remainder by twice the length of the parallel bar, for a divisor. And if the above dividend be divided by this divisor, and the quotient subtracted from the length of the parallel bar, the remainder is the length of the radius rod.

Ex.—Given the radius of the beam 10 feet, the length of the parallel bar 6 feet, and the length of the stroke 6 feet, to find the length of the radius rod.

12 = twice the length of parallel bar.
10 = radius of the beam.
—
2
9 = square of half the length of stroke.
—
18 = dividend mentioned in the rule.

100 = square of the radius of the beam.
9 = do. half length of stroke.
—
91 the square root of which is 9·539.

10 = radius of the beam.
9·539
—
·461

·461
12 = twice the length of parallel bar.

5·532 = divisor mentioned in rule.

18 divided by 5·532 = 3·25.

6·00 = length of parallel bar.
3·25

2·75 = length of radius rod required.

In steam-boat engines, a parallel motion may be put on so as to make the air bucket rod, the piston rod, and the feed pump rods, all move in the same kind of a line, that is, approximating to a vertical straight line, by fixing the air bucket cross-head into the point F in the back link (see Fig. 6), and by attaching the feed pumps to this cross-head. Mr. Dodds has built a steam-boat engine on this principle, which is the first that I have ever heard of.

Steam-boat Wheels.—Large wheels should always be preferred to small ones. They strike the water in a direction which is most favourable: not only so, but the direction in which they leave the water is more favourable than that of small wheels. Large wheels move with a greater velocity, and therefore they throw the back-water off with a greater facility. Small wheels, for want of sufficient velocity and a proper direction, carry nearly as much water round to the fore side of the paddle wheels as they discharge at the back side. But it has been objected by some, that large wheels cannot be driven by an engine so well as small ones; but this is a mistake, for the engine may be made to make her strokes in the same time with a large wheel as with a small one. It is only necessary to make the paddle proportionably less; and if the length of the paddle arm, multiplied by the area of the paddle, be the same in both cases, the engine will perform her stroke in the same time, whether the wheel be large or small. If the fluid be at rest, then the difference between the velocity of the paddles and the velocity of the boat is equal to the velocity with which the paddles act upon the water. Hence, when these velocities are equal, the paddles will act with no force to impel the boat; and if the paddles were to move slower than the boat, then the boat would be retarded by them. The velocity of the paddles should be about $1\frac{1}{2}$ times the velocity of the boat; for, with this excess of velocity, the effective power is greatest.

Page 135, line 1, for one inch, read one foot; line 4, for 4 feet $11\frac{1}{2}$ inches, read 4 feet 6 inches; and line 12, for Fig. 9, read Fig. 6.

At page 139, lines 27, 28, 29, and 30, in a few impressions of this work, for $51\frac{1}{2}^\circ + 7\frac{1}{2}^\circ$, &c. read $51\frac{1}{2}^\circ - 7\frac{1}{2}^\circ = 44^\circ$, the angle which the connecting rod makes with the crank, the natural sine of which is $\cdot 69466 =$ the effective leverage; and $\cdot 69466 \times 12 = 8\cdot 33592$ inches.

To find the Shape of Boiler Plates for Boilers which have their Ends of a semi-spherical Form, spherical, or any Segment of a Sphere.

The plates, when flat, should be of such a form, that when bended, they will all fit each other in straight edges, which evidently cannot be the case if the sides are straight lines.

Find the circumference of the boiler end, and divide it by the number of plates, and the quotient will give the breadth of each plate; then find the circumference at any other part of the globular end, and divide it by the same number of plates, and the quotient will give the breadth of each plate at that point. Then let A B (Fig. 13 to *Steam Engine)* represent the breadth of the plate at the top; and from any point F in G C set off F D and F E, each equal to half the breadth of the plate at that point. Then, through the three points A, D, and C, describe the circular arc A D C; and through the three points B, E, and C, describe the circular arc B E C; and this will be the form of the sides of the plate. From C, with the radius C A, describe the arc A G B, which will be the form of the top end of the plate; and if each plate of this form be bended so as to form the boiler end, they will all form themselves together in straight edges.

Note,—No overlap is here allowed for; but it must be taken into account in practice.

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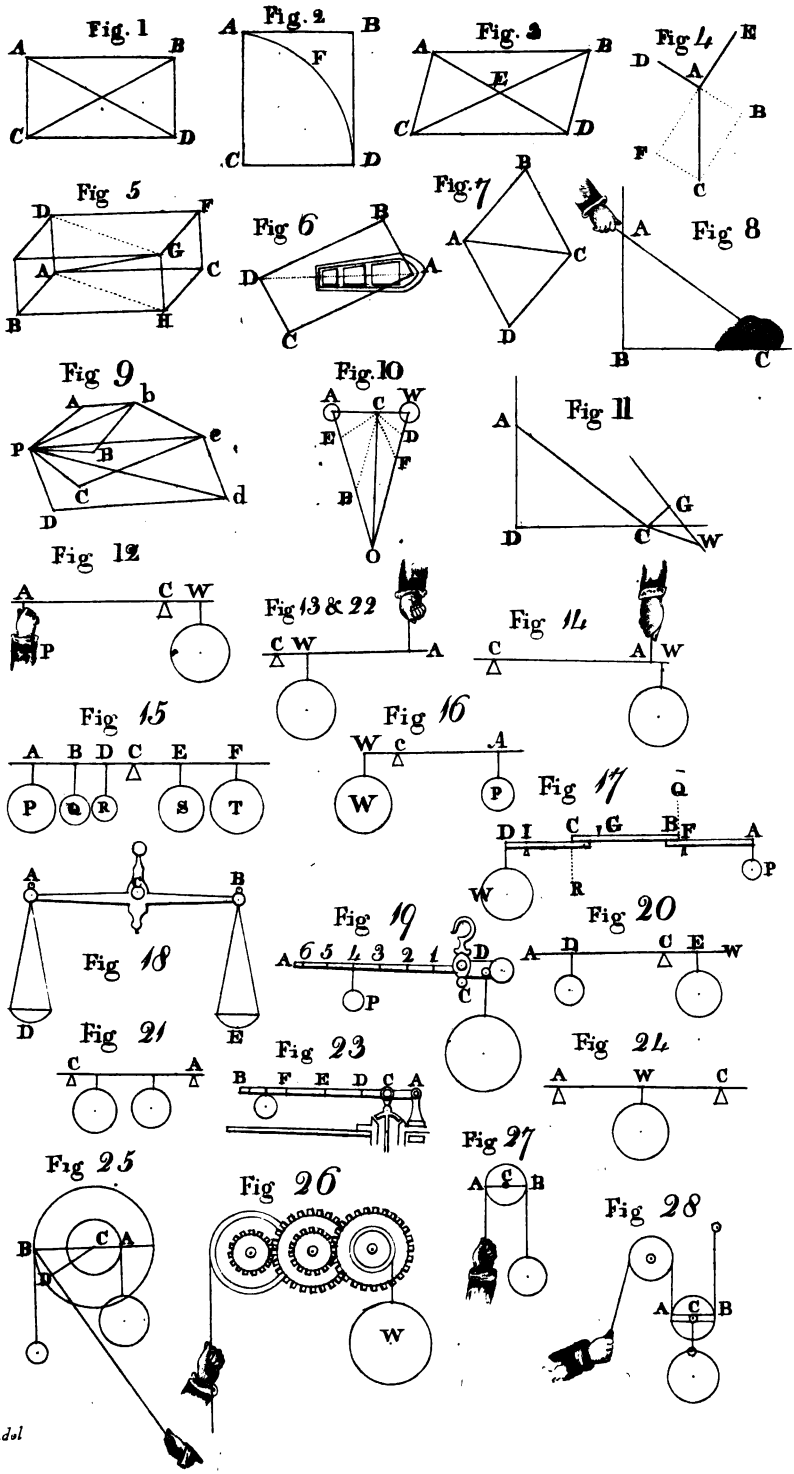
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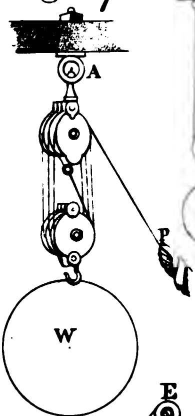

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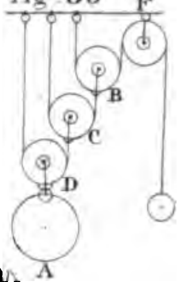

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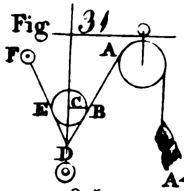

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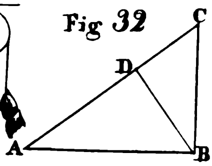

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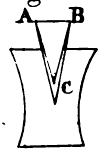

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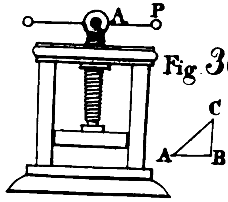

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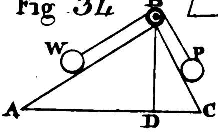

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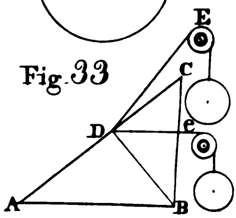

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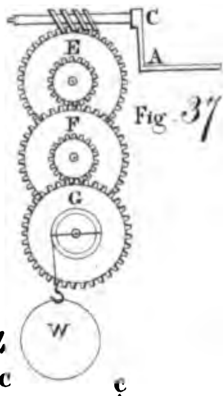

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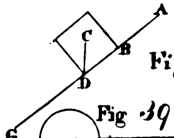

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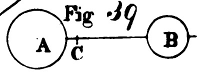

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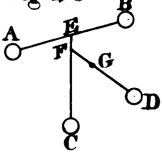

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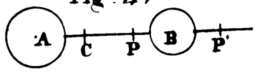

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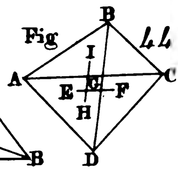

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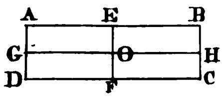

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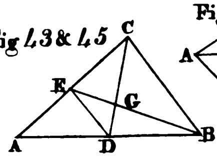

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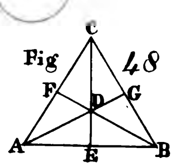

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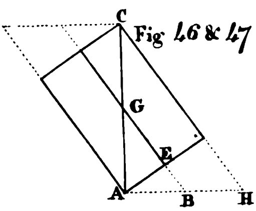

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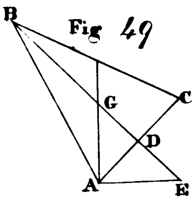

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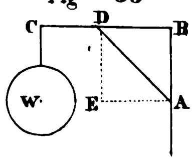

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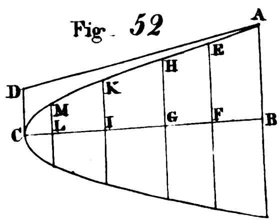

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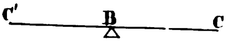

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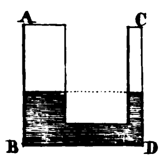

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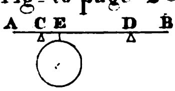

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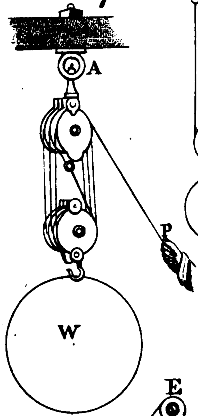

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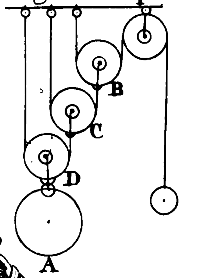

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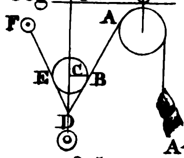

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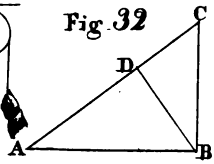

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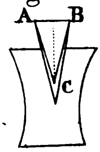

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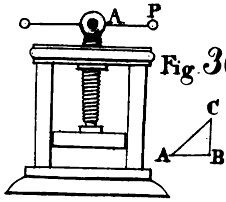

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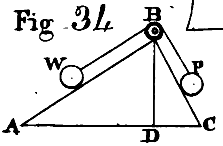

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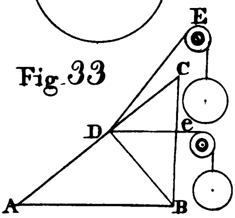

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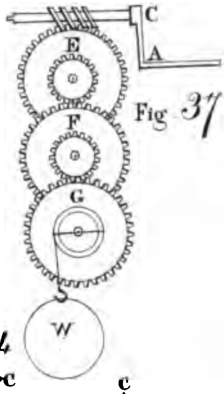

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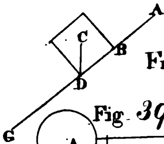

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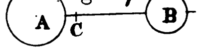

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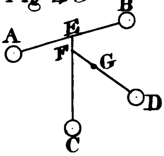
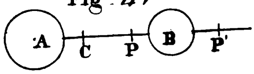


Fig. 41



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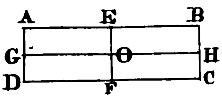
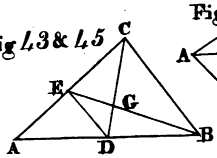


Fig 43 & 45



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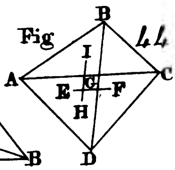


Fig 48

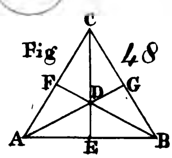
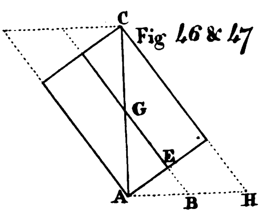


Fig 46 & 47



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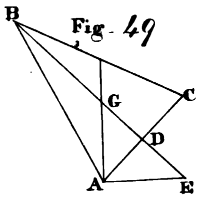

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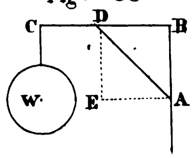


Fig. 52

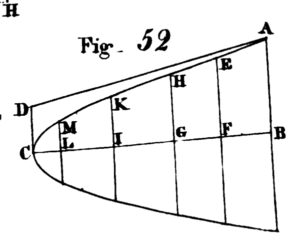


Fig. 51

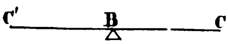


Fig. to Hydrostatics

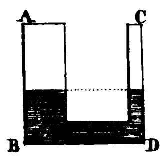
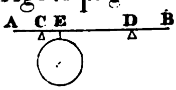


Fig. to page 25



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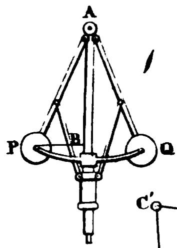
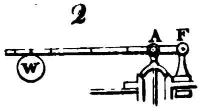
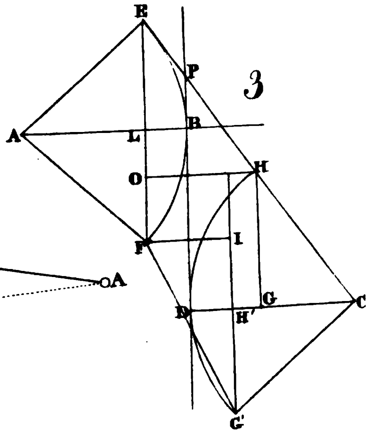
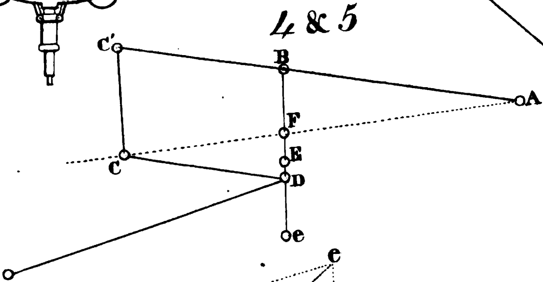
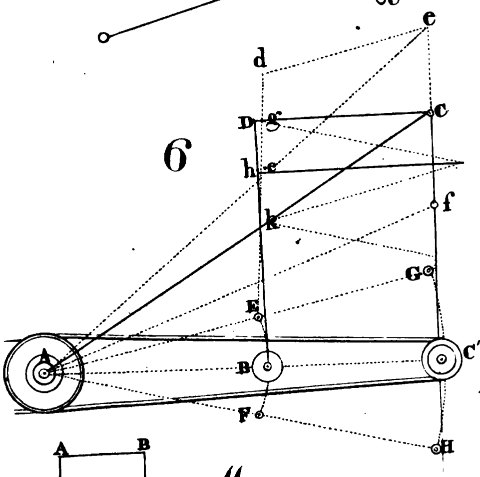
Figures to Steam Engines

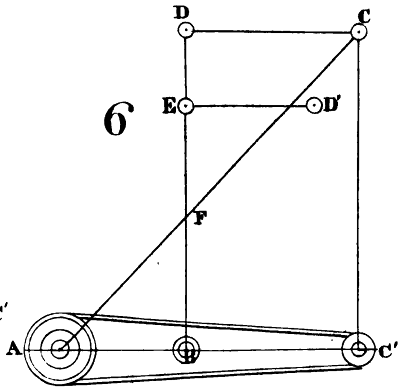


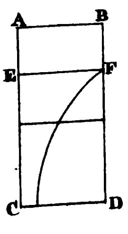


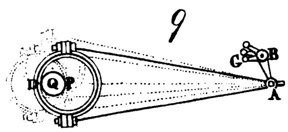


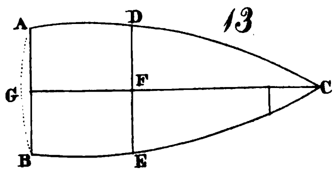


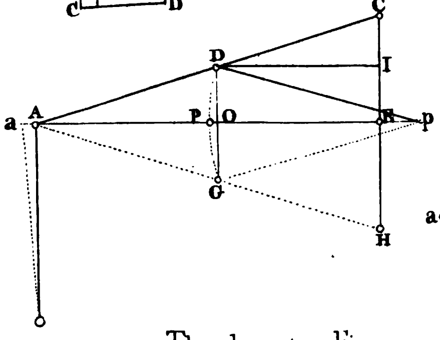


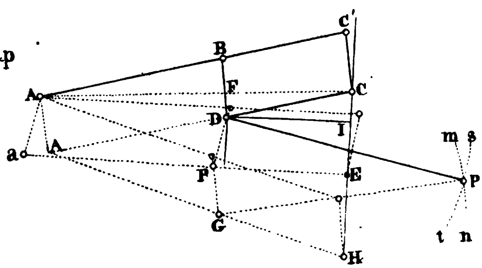












The above two Figures answer to Geometrical Constructions and Fig. 7 & 8

J. Dodd ...